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Sarvesh Kumar Dubey
Department of Agricultural Statistics, A.N.D.U.A.T, Ayodhya, Uttar Pradesh, India

Manish Kumar
Department of Agricultural Statistics, A.N.D.U.A.T, Ayodhya, Uttar Pradesh, India

VN Rai
Department of Agricultural Statistics, A.N.D.U.A.T, Ayodhya, Uttar Pradesh, India

Ashish Tiwari
Department of Agricultural Statistics, A.N.D.U.A.T, Ayodhya, Uttar Pradesh, India

Corresponding Author
Manish Kumar
Department of Agricultural Statistics, A.N.D.U.A.T, Ayodhya, Uttar Pradesh, India

A composite class of estimators for a finite population mean using two auxiliary variables in two-phase sampling

Sarvesh Kumar Dubey, Manish Kumar, VN Rai and Ashish Tiwari

Abstract
A composite class of estimators has been formulated in this paper for estimating the mean of a finite population using two auxiliary variables under two-phase sampling design. The mean square errors (MSEs) of the proposed class and the other well-known pre-existing estimators have been computed to the first order of approximation. The precision of the proposed class of estimators has been compared with various pre-existing estimators. The theoretical results have been accompanied with an empirical analysis using real population datasets, and the percent relative efficiencies (PREs) have been computed for the various suggested estimators.

Keywords: Study variable, auxiliary variable, two-phase sampling, mean square error, percent relative efficiency

1. Introduction
In the theory of sample surveys, the problem of estimation of population parameters (such as population mean and population variance) has a significant role. For instance, small area estimation (SAE) problems, crop cutting experiments, employment and unemployment surveys, household surveys, health surveys, labor force surveys, and so on. In sample surveys, the information regarding a population is conveyed by considering a part of the population (known as sample) instead of dealing with the entire units of the population. It provides the information accurately and in short duration of time, provided that it is organized in a well-defined manner. Also, it is a cost effective method as compared to the complete enumeration.
For estimating the population mean of a study variable, ratio estimator was developed by (Cochran, 1940) [1] by utilizing the prior information on an auxiliary variable, which is positively correlated with the study variable. However, in some cases, the prior information on auxiliary variable is not available, and in that case it is desirable to use double (or two-phase) sampling design, which was initially introduced by (Neyman, 1938) [2].
The two-phase sampling design for estimation of mean involves two steps: (i) selecting a preliminary large sample (known as first-phase sample) of size n' from a population of size N to measure the mean of auxiliary variable, and (ii) selecting a subsample (known as second-phase sample) of size n from the preliminary sample of size n' for measuring the means of the study variable and the auxiliary variable.
The problem of estimation of mean in two-phase sampling has been dealt by various scientists and statisticians. In recent years, some remarkable contributions in this direction have been made, for instance (Singh and Vishwakarma, 2007) [3], (Vishwakarma and Kumar, 2014) [4], (Vishwakarma and Kumar, 2015) [5], (Vishwakarma and Kumar, 2016) [6], (Kumar and Vishwakarma, 2017) [7], (Dubey *et al.*, 2020) [8], and (Kumar and Tiwari, 2021) [9].

2. Some pre-existing estimators
(Sukhatme, 1962) [10] developed the following ratio estimator under two-phase sampling for the estimation of population mean $\bar{Y}$ of the study variable Y:

$$\bar{y}_R^d = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right) \tag{1}$$

Where $\bar{x}'=\sum_{i=1}^{n'}X_i/n'$ denotes the first-phase sample mean of the auxiliary variable *X*. Also, $\bar{y}=\sum_{i=1}^{n}Y_i/n$ and $\bar{x}=\sum_{i=1}^{n}X_i/n$ denote, respectively, the second-phase sample means of the variables *Y* and *X*.

(Srivastava, 1970) [11] utilized a scalar quantity α and suggested the following ratio estimator for $\bar{Y}$ under two-phase sampling:

$$\bar{y}_{ds}=\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha} \quad (2)$$

(Chand, 1975) [12] utilized two auxiliary variables *X* and *Z*, such that *X* is closely related to *Y* as compared to *Z* (i.e., $\rho_{YX}>\rho_{YZ}>0$), and developed the following chain ratio type estimator for $\bar{Y}$:

$$\bar{y}_R^{dc}=\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right) \quad (3)$$

Where $\bar{Z}=\sum_{i=1}^{N}Z_i/N$ and $\bar{z}'=\sum_{i=1}^{n'}Z_i/n'$ denote, respectively, the population mean and the first phase sample mean of the auxiliary variable *Z*.

(Mukerjee *et al*., 1987) [23] defined the following regression estimator for $\bar{Y}$ under two-phase sampling:

$$\bar{y}_{MEA}^{d}=\bar{y}+b_{yx}(\bar{x}'-\bar{x})+b_{yz}(\bar{z}'-\bar{z}) \quad (4)$$

Where b_{yx} and b_{yz} denote respectively, the sample regression coefficient of *Y* on *X*, and of *Y* on *Z*. Also, $\bar{z}=\sum_{i=1}^{n}Z_i/n$ represents the second-phase sample mean of the auxiliary variable *Z*.

(Singh and Upadhyaya, 1995) [14] suggested the following class of modified chain-type estimators for $\bar{Y}$ by using information on known population coefficient of variation of variable Z:

$$\bar{y}_{SU}^{dc}=\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}+C_Z}{\bar{z}'+C_Z}\right)^{\alpha} \quad (5)$$

Where α is an unknown constant. (Singh *et al*., 2007) [15] utilized the information on the correlation coefficient between the variable *X* and *Z*, and defined the following chain ratio-type estimator for $\bar{Y}$:

$$\bar{y}_{SEA}=\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}+\rho_{XZ}}{\bar{z}'+\rho_{XZ}}\right) \quad (6)$$

(Singh and Choudhury, 2012) [16] suggested the following exponential chain ratio-type estimator for $\bar{Y}$:

$$\bar{y}_{\mathrm{Re}}^{dc}=\bar{y}\exp\left\{\frac{(\bar{x}'/\bar{z}')\bar{Z}-\bar{x}}{(\bar{x}'/\bar{z}')\bar{Z}+\bar{x}}\right\}. \quad (7)$$

(Singh and Majhi, 2014) [17] suggested the following exponential type estimator for $\bar{Y}$:

$$\bar{y}_{SM}=\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\exp\left(\frac{\bar{Z}-\bar{z}'}{\bar{Z}+\bar{z}'}\right). \quad (8)$$

To the first order of approximation, the mean square errors (MSEs) of various estimators mentioned earlier are as follows:

$$MSE(\bar{y}_R^d)=\bar{Y}^2\left\{f_1C_Y^2+f_3\left(C_X^2-2\rho_{YX}C_YC_X\right)\right\} \quad (9)$$

$$MSE(\bar{y}_{ds})=\bar{Y}^2\left\{f_1C_Y^2+f_3\left(\alpha^2C_X^2-2\alpha\rho_{YX}C_YC_X\right)\right\} \quad (10)$$

$$MSE(\bar{y}_R^{dc})=\bar{Y}^2\left\{\begin{array}{l}f_1C_Y^2+f_3C_X^2+f_2C_Z^2\\-2f_3\rho_{YX}C_YC_X-2f_2\rho_{YZ}C_YC_Z\end{array}\right\} \quad (11)$$

$$MSE(\bar{y}_{MEA}^{d})=\bar{Y}^2\left\{f_1C_Y^2-f_3C_Y^2\left(\begin{array}{l}\rho_{YX}^2+\rho_{YZ}^2\\-2\rho_{XZ}\rho_{YX}\rho_{YZ}\end{array}\right)\right\} \quad (12)$$

$$MSE(\bar{y}_{SU}^{dc})=\bar{Y}^2\left\{\begin{array}{l}f_1C_Y^2+f_2\left(\alpha^2\zeta^2C_Z^2-2\alpha\zeta\rho_{YZ}C_YC_Z\right)\\+f_3\left(C_X^2-2\rho_{YX}C_YC_X\right)\end{array}\right\} \quad (13)$$

$$MSE(\bar{y}_{SEA})=\bar{Y}^2\left\{\begin{array}{l}f_1C_Y^2+f_3C_X^2+\psi^2f_2C_Z^2\\-2f_3\rho_{YX}C_YC_X-2\psi f_2\rho_{YZ}C_YC_Z\end{array}\right\} \quad (14)$$

$$MSE(\bar{y}_{\mathrm{Re}}^{dc})=\bar{Y}^2\left\{\begin{array}{l}f_1C_Y^2+\frac{1}{4}\left(f_3C_X^2+f_2C_Z^2\right)\\-\left(f_3\rho_{YX}C_YC_X+f_2\rho_{YZ}C_YC_Z\right)\end{array}\right\} \quad (15)$$

$$MSE(\bar{y}_{SM})=\bar{Y}^2\left\{\begin{array}{l}f_1C_Y^2+f_3\left(C_X^2-2\rho_{YX}C_YC_X\right)\\+\frac{1}{4}\left(f_2C_Z^2-4f_2\rho_{YZ}C_YC_Z\right)\end{array}\right\} \quad (16)$$

Moreover, the minimum attainable MSEs of the estimators $\bar{y}_{ds}$ and $\bar{y}_{SU}^{dc}$ are given, respectively, by

$$MSE(\bar{y}_{ds})_{\min}=\bar{Y}^2C_Y^2\left(f_1-f_3\rho_{YX}^2\right) \quad (17)$$

$$MSE(\bar{y}_{SU}^{dc})_{\min}=\bar{Y}^2\left\{\begin{array}{l}\left(f_1-f_2\rho_{YZ}^2\right)C_Y^2\\+f_3\left(C_X^2-2\rho_{YX}C_YC_X\right)\end{array}\right\} \quad (18)$$

The notations used above are defined as follows:

$$f_1=\left(\frac{1}{n}-\frac{1}{N}\right),$$

$$f_2=\left(\frac{1}{n'}-\frac{1}{N}\right),$$

$$f_3=f_1-f_2=\left(\frac{1}{n}-\frac{1}{n'}\right),$$

$$\psi = \frac{\bar{Z}}{(\bar{Z} + \rho_{XZ})},$$

$$\zeta = \frac{\bar{Z}}{(\bar{Z} + C_Z)}$$

$$C_Y^2 = \frac{S_Y^2}{\bar{Y}^2},$$

$$C_X^2 = \frac{S_X^2}{\bar{X}^2},$$

$$C_Z^2 = \frac{S_Z^2}{\bar{Z}^2},$$

$$\rho_{YX} = \frac{S_{YX}}{S_Y S_X},$$

$$\rho_{YZ} = \frac{S_{YZ}}{S_Y S_Z},$$

$$\rho_{XZ} = \frac{S_{XZ}}{S_X S_Z},$$

$$S_Y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2,$$

$$S_X^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2,$$

$$S_Z^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(Z_i - \bar{Z}\right)^2,$$

$$S_{YX} = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right),$$

$$S_{YZ} = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)\left(Z_i - \bar{Z}\right),$$

$$S_{XZ} = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Z_i - \bar{Z}\right).$$

3. Proposed class of estimators

Motivated by the work of (Singh *et al*., 2007) [15], we propose the following composite class of estimators for population mean $\bar{Y}$ under two-phase sampling.

$$T = \alpha\,\bar{y} + (1-\alpha)\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z} + \rho_{XZ}}{\bar{z}' + \rho_{XZ}}\right), \qquad (19)$$

Where α is a scalar quantity, which may be either a real number or a function of some known parameters of the auxiliary variable (s).

Remark 3.1 For $\alpha = 0$, the proposed class T in (19) reduces to the (Singh *et al*., 2007) [15] estimator $\bar{y}_{SEA}$ as given in (6). Also, For $\alpha = 1$, the proposed class T in (19) reduces to the sample mean $\bar{y}$, which is an unbiased estimator of the population mean $\bar{Y}$.

4. MSE of the proposed class

To obtain the MSE of the proposed class T, we consider
$\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$, $\bar{x}' = \bar{X}(1+e_1')$,
$\bar{z}' = \bar{Z}(1+e_2')$.

Also, we have

$$\left.\begin{aligned}
&E(e_0)=E(e_1)=E(e_1')=E(e_2')=0,\\
&E(e_0^2)=f_1C_Y^2,\ E(e_1^2)=f_1C_X^2,\ E(e_1'^2)=f_2C_X^2,\ E(e_2'^2)=f_2C_Z^2,\\
&E(e_0e_1)=f_1\rho_{YX}C_YC_X,\ E(e_0e_1')=f_2\rho_{YX}C_YC_X,\ E(e_0e_2')=f_2\rho_{YZ}C_YC_Z,\\
&E(e_1e_1')=f_2C_X^2,\ E(e_1e_2')=f_2\rho_{XZ}C_XC_Z,\ E(e_1'e_2')=f_2\rho_{XZ}C_XC_Z.
\end{aligned}\right\} \qquad (20)$$

Now, expressing T in terms of e_0, e_1, e_1' and e_2', we have

$$T = \left[\begin{aligned}&\alpha\bar{Y}(1+e_0)\\ &+(1-\alpha)\bar{Y}(1+e_0)(1+e_1')(1+e_1)^{-1}(1+\psi e_2')^{-1}\end{aligned}\right] \qquad (21)$$

Multiplying out and retaining the first order error terms in equation (21), we have

$$T - \bar{Y} = \bar{Y}\left[e_0 - (1-\alpha)\left(e_1 - e_1' + \psi e_2'\right)\right] \qquad (22)$$

Now, squaring both sides of (22), we have

$$(T-\bar{Y})^2 = \bar{Y}^2\left[\begin{aligned}&e_0^2 - 2(1-\alpha)(e_0e_1 - e_0e_1' + \psi\, e_0e_2')\\ &+(1-\alpha)^2\left(\begin{aligned}&e_1^2 + e_1'^2 - 2e_1e_1' + \psi^2 e_2'^2\\ &+2\psi\, e_1e_2' - 2\psi\, e_1'e_2'\end{aligned}\right)\end{aligned}\right] \qquad (23)$$

Taking the expectation in (23) and using the results of (20), we obtain the MSE of the proposed class T to the first order of approximation as

$$\begin{aligned}MSE(T) &= \bar{Y}^2[\,f_1C_Y^2 - 2(1-\alpha)(\,f_3\rho_{YX}C_YC_X + \psi\ f_2\rho_{YZ}C_YC_Z) \\ &\quad + (1-\alpha)^2(\,f_3C_X^2 + \psi^2 f_2C_Z^2)]\end{aligned} \qquad (24)$$

The MSE of the proposed class T at (24) is minimized for

$$\alpha = 1 - \left(\frac{f_3\rho_{YX}C_YC_X + \psi\ f_2\rho_{YZ}C_YC_Z}{f_3C_X^2 + \psi^2 f_2C_Z^2}\right) \qquad (25)$$

and hence the minimum attainable MSE of T is given by

$$MSE(T)_{\min} = \bar{Y}^2 C_Y^2\left[f_1 - \frac{(f_3\rho_{YX}C_X + \psi f_2\rho_{YZ}C_Z)^2}{f_3C_X^2 + \psi^2 f_2C_Z^2}\right]. \quad (26)$$

Hence, we establish the following theorem.

Theorem 4.1 To the first order approximation,

$$MSE(T) \ge \bar{Y}^2 C_Y^2\left[f_1 - \frac{(f_3\rho_{YX}C_X + \psi f_2\rho_{YZ}C_Z)^2}{f_3C_X^2 + \psi^2 f_2C_Z^2}\right] \quad (27)$$

With equality holding if

$$\alpha = 1 - \left(\frac{f_3\rho_{YX}C_YC_X + \psi f_2\rho_{YZ}C_YC_Z}{f_3C_X^2 + \psi^2 f_2C_Z^2}\right)$$

5. Efficiency comparisons

The variance of sample mean $\bar{y}$ under simple random sampling without replacement (SRSWOR) scheme is given by

$$Var(\bar{y}) = f_1\bar{Y}^2C_Y^2 \quad (28)$$

For making efficiency comparisons of the proposed class T with the pre-existing estimators, we obtain the following conditions on using equations (9) to (16), (24), and (28):

(i) $MSE(T) < Var(\bar{y})$ if

$$C_Y < \frac{1}{2}(1-\alpha)\left(\frac{f_3C_X^2 + \psi^2 f_2C_Z^2}{f_3\rho_{YX}C_X + \psi f_2\rho_{YZ}C_Z}\right) \quad (29)$$

(ii) $MSE(T) < MSE(\bar{y}_R^d)$ if

$$C_Y < \frac{1}{2}\left[\frac{(1-\alpha)^2\psi^2 f_2C_Z^2 - \alpha(2-\alpha)f_3C_X^2 + 2f_3\rho_{YX}C_YC_X}{(1-\alpha)f_3\rho_{YX}C_X + \psi f_2\rho_{YZ}C_Z}\right] \quad (30)$$

(iii) $MSE(T) < MSE(\bar{y}_{ds})$ if

$$C_Y < \frac{1}{2}\left[\frac{(1-\alpha)^2\psi^2 f_2C_Z^2 + (1-2\alpha)f_3C_X^2}{(1-2\alpha)f_3\rho_{YX}C_X + \psi(1-\alpha)f_2\rho_{YZ}C_Z}\right] \quad (31)$$

(iv) $MSE(T) < MSE(\bar{y}_R^{dc})$ if

$$C_Y < \frac{1}{2}\left[\frac{\alpha(2-\alpha)(f_3C_X^2 + f_2C_Z^2)}{\alpha f_3\rho_{YX}C_X + \{1-(1-\alpha)\psi\} f_2\rho_{YZ}C_Z}\right] \quad (32)$$

(v) $MSE(T) < MSE(\bar{y}_{MEA}^d)$ if

$$C_Y < \frac{1}{2}\left[\frac{(1-\alpha)^2(f_3C_X^2 + \psi^2 f_2C_Z^2)}{(1-\alpha)(f_3\rho_{YX}C_X + \psi f_2\rho_{YZ}C_Z) - (1/2)(f_3C_Y)(\rho_{YX}^2 + \rho_{YZ}^2 - 2\rho_{XZ}\rho_{YX}\rho_{YZ})}\right] \quad (33)$$

(vi) $MSE(T) < MSE(\bar{y}_{SU}^{dc})$ if

$$C_Y < \frac{1}{2}\left[\frac{\{f_2(1-\alpha)^2 - f_3\alpha^2\zeta^2\}C_Z^2 + \alpha(2-\alpha)f_3C_X^2}{(1-2\alpha)f_3\rho_{YX}C_X - \{\alpha\zeta - (1-\alpha)\psi\} f_2\rho_{YZ}C_Z}\right] \quad (34)$$

(vii) $MSE(T) < MSE(\bar{y}_{SEA})$ if

$$C_Y < \frac{1}{2}\left[\frac{(2-\alpha)(f_3C_X^2 + \psi^2 f_2C_Z^2)}{f_3\rho_{YX}C_X + \psi f_2\rho_{YZ}C_Z}\right] \quad (35)$$

(viii) $MSE(T) < MSE(\bar{y}_{\mathrm{Re}}^{dc})$ if

$$C_Y < \frac{1}{2}\left[\frac{(\alpha^2 - 2\alpha + 3/4)f_3C_X^2 + (\psi^2\alpha^2 - 2\alpha\psi^2 + \psi^2 - 1/4)f_2C_Z^2}{(1/2-\alpha)f_3\rho_{YX}C_X - (\alpha\psi - \psi + 1/2)f_2\rho_{YZ}C_Z}\right] \quad (36)$$

(ix) $MSE(T) < MSE(\bar{y}_{SM})$ if

$$C_Y < \frac{1}{2}\left[\frac{(2-\alpha)\alpha f_3C_X^2 - (\psi^2\alpha^2 - 2\alpha\psi^2 + \psi^2 - 1/4)f_2C_Z^2}{\alpha f_3\rho_{YX}C_X + (\alpha\psi - \psi + 1/2)f_2\rho_{YZ}C_Z}\right] \quad (37)$$

6. Empirical analysis

In order to examine the efficiency of the proposed class T with respect to the other existing estimators, we have considered five real population datasets. The description of the populations, along with the respective parameters, is mentioned below:

Population I: (Anderson, 1958) [18]
Y: Head length of second son,
X: Head length of first son,
Z: Head breadth of first son,

$$N = 25, n' = 10, n = 7,$$
$$\bar{Y} = 183.84, \bar{X} = 185.72, \bar{Z} = 151.12,$$
$$\rho_{YX} = 0.7108, \rho_{YZ} = 0.6932, \rho_{XZ} = 0.7346,$$
$$C_Y = 0.0546, C_X = 0.0526, C_Z = 0.0488.$$

Population II: (Handique, 2012) [19]
Y: Forest timber volume in cubic meter (Cum) in 0.1 ha sample plot,
X: Average tree height in the sample plot in meter (m),
Z: Average crown diameter in the sample plot in meter (m),

$$N = 2500, n' = 200, n = 25,$$
$$\bar{Y} = 4.63, \bar{X} = 21.09, \bar{Z} = 13.55,$$

$$\rho_{YX} = 0.79, \rho_{YZ} = 0.72, \rho_{XZ} = 0.66,$$
$$C_Y = 0.95, C_X = 0.98, C_Z = 0.64.$$

Population III- (Murthy, 1967) [20]
Y: Area under wheat in 1964,
X: Area under wheat in 1963,
Z: Cultivated area in 1961,

$$\text{N} = 34, n' = 10, n = 7,$$
$$\bar{Y} = 199.44, \bar{X} = 208.89, \bar{Z} = 747.59,$$
$$\rho_{YX} = 0.9801, \rho_{YZ} = 0.9043, \rho_{XZ} = 0.9097,$$
$$C_Y^2 = 0.5673, C_X^2 = 0.5191, C_Z^2 = 0.3527.$$

Population IV- (Sukhatme and Chand, 1977) [21]
Y: Apple trees of bearing age in 1964,
X: Bushels of apples harvested in 1964,
Z: Bushels of apples harvested in 1959,

$$\text{N} = 200, n' = 30, n = 20,$$
$$\bar{Y} = 1031.82, \bar{X} = 2934.58, \bar{Z} = 3651.49,$$
$$\rho_{YX} = 0.93, \rho_{YZ} = 0.77, \rho_{XZ} = 0.84,$$
$$C_Y^2 = 2.55280, C_X^2 = 4.02504, C_Z^2 = 2.09379.$$

Population V- (Cochran, 1977) [22]
Y: Number of 'placebo' children,
X: Number of paralytic polio cases in the 'placebo' group,
Z.: Number of paralytic polio cases in the 'not inoculated' group,

N = 34, n' = 15, n = 10,
$\bar{Y}$ = 4.92, $\bar{X}$ = 2.59, $\bar{Z}$ =2.91,
ρ_{YX} = 0.7326, ρ_{YZ} = 0.6430, ρ_{XZ} =0.6837,
C_Y^2 = 1.0248, C_X^2 = 1.5175, C_Z^2 = 1.1492.

The percent relative efficiencies (PREs) of various estimators of $\bar{Y}$ have been obtained with respect to the sample mean $\bar{y}$, and the findings are presented in Table 1. The PREs are computed by using the formula:

$$PRE(\phi, \bar{y}) = \frac{Var(\bar{y})}{MSE(\phi)} \times 100,$$

Where

$\phi = \bar{y}, \bar{y}_R^d, \bar{y}_{ds}, \bar{y}_R^{dc}, \bar{y}_{MEA}^d, \bar{y}_{SU}^{dc}, \bar{y}_{SEA}, \bar{y}_{\text{Re}}^{dc}, \bar{y}_{SM}, T$.

Table 1: PREs of various estimators of $\bar{Y}$ with respect to the sample mean $\bar{y}$

Estimator	Population I	Population II	Population III	Population IV	Population V
$\bar{y}$	100	100	100	100	100
$\bar{y}_R^d$	122.54	200.01	156.91	139.09	116.65
$\bar{y}_{ds}$	126.66	223.02	156.96	147.13	133.95
$\bar{y}_R^{dc}$	178.82	227.27	730.81	279.93	136.91
$\bar{y}_{MEA}^d$	112.25	152.95	106.68	110.42	116.89
$\bar{y}_{SU}^{dc}$	186.65	227.40	778.27	289.32	156.47
$\bar{y}_{SEA}$	179.14	227.04	730.07	279.96	150.75
$\bar{y}_{\text{Re}}^{dc}$	176.54	212.00	259.55	247.82	184.36
$\bar{y}_{SM}$	175.09	218.91	344.39	244.51	154.82
T	196.26	253.86	762.95	322.93	187.34

7. Results and Conclusion

Table 1 exhibits the following results:

1. In all the five populations, the proposed class *T* has the maximum PREs as compared to the sample mean ($\bar{y}$) and the other pre-existing estimators except in population III, in which Singh and Upadhyaya (1995) [14] estimator $\bar{y}_{SU}^{dc}$ has the maximum PRE.
2. In the first four populations, the PREs of Chand (1975) [12] estimator $\bar{y}_R^{dc}$ are approximately the same as that of the Singh *et al*. (2007) [15] estimator $\bar{y}_{SEA}$.
3. In all the five populations, the PREs of the sample mean ($\bar{y}$) are less as compared to the PREs of the proposed class *T*, as well as that of the other existing estimators.

Hence, from the above results, we conclude that the proposed class *T* outperforms the sample mean ($\bar{y}$) as well as the other pre-existing estimators for the estimation of mean $\bar{Y}$ of the study variable Y. Moreover, it is also revealed that the estimators which involve auxiliary variable(s), are more efficient as compared to the sample mean ($\bar{y}$). So, the information on auxiliary variable(s) is preferably utilized at the estimation stage.

8. References

1. Cochran WG. The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce. The Journal of Agricultural Science. 1940;30:262-275.
2. Neyman J. Contribution to the theory of sampling human populations. Journal of the American Statistical Association. 1938;33:101-116.
3. Singh HP, Vishwakarma GK. Modified exponential ratio and product estimators for finite population mean in double sampling. Austrian Journal of Statistics. 2007;36(3):217-225.
4. Vishwakarma GK, Kumar M. An improved class of chain ratio-product type estimators in two-phase sampling

using two auxiliary variables. Journal of Probability and Statistics, 2014, 1-6.
5. Vishwakarma GK, Kumar M. An efficient class of estimators for the mean of a finite population in two-phase sampling using multi-auxiliary variates. Communications in Mathematics and Statistics. 2015;3:477-489.
6. Vishwakarma GK, Kumar M. A new approach to mean estimation using two auxiliary variates in two-phase sampling. Proceedings of the National Academy of Sciences, India, Section A: Physical Sciences. 2016;86(1):33-39.
7. Kumar M, Vishwakarma GK. Estimation of mean in double sampling using exponential technique on multi-auxiliary variates. Communications in Mathematics and Statistics. 2017;5:429-445.
8. Dubey SK, Sisodia BVS, Sharma MK. Some transformed and composite chain ratio-type estimators using two auxiliary variables. Journal of the Indian Society of Agricultural Statistics. 2020;74(1):17-22.
9. Kumar M, Tiwari A. A generalized class of estimators of population mean in two-phase sampling using two auxiliary variables. International Journal of Mathematics and Statistics. 2021;22(3):18-27.
10. Sukhatme BV. Some ratio-type estimators in two-phase sampling. Journal of the American Statistical Association. 1962;57:628-632.
11. Srivastava SK. A two-phase sampling estimator in sample surveys. Australian Journal of Statistics. 1970;12(1):23-27.
12. Chand L. Some ratio type estimators based on two or more auxiliary variables. Ph.D. dissertation. Iowa State University, Ames, Iowa (USA), 1975.
13. Sahoo RK, Sabat AK, Nayak RK, Sahoo LN. On a method of estimating variance of the product estimator. International Journal of Statistics and Applied Mathematics. 2021;6(6):16-23.
14. Singh GN, Upadhyaya LN. A class of modified chain type estimators using two auxiliary variables in two phase sampling. Metron. 1995;LIII:117-125.
15. Singh R, Chauhan P, Sawan N. A family of estimators for estimating population mean using known correlation coefficient in two phase sampling. Statistics in Transition. 2007;8(1):89-96.
16. Singh BK, Choudhury S. Exponential chain ratio and product type estimators for finite population mean under double sampling scheme. Global Journal of Science Frontier Research: Mathematics and Decision Sciences. 2012;12(6):13-24.
17. Singh GN, Majhi D. Some chain-type exponential estimators of population mean in two-phase sampling. Statistics in Transition. 2014;15(2):221-230.
18. Anderson TW. An introduction to multivariate statistical analysis. New York: John Wiley and Sons, 1958.
19. Handique BK. A class of regression-cum-ratio estimators in two-phase sampling for utilizing information from high resolution satellite data. ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences. 2012;I-4:71-76.
20. Murthy MN. Sampling theory and methods. Calcutta, India: Statistical Publishing Society, 1967.
21. Sukhatme BV, Chand L. Multivariate ratio-type estimators. Proceedings of American Statistical Association, Social Statistics Section, 1977, 927-931.
22. Cochran WG. Sampling techniques, 3rd edn. New York: John Wiley and Sons, 1977.
23. Mukerjee R, Rao TJ, Vijayan K. Regression type estimators using multiple auxiliary information. Australian Journal of Statistics. 1987;29:244-254.