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Use of double sampling to estimate the finite population mean

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Abstract
We have discussed the topic of estimating the finite population mean in double sampling in this paper. In this article, Singh and Tailor (2005) propose a ratio-cum-product type estimator for double sampling. Up to the first degree of approximation, biases and mean squared errors (MSEs) were determined. Existing estimators have been compared to the recommended estimators. An empirical research was carried out to assess how well the recommended estimators performed.

Keywords: Population mean, ratio-cum-product type estimator, correlation coefficient, bias, mean squared error

Introduction

The finite population mean of the auxiliary variate is assumed by ratio, product, and regression type estimators, however in many actual circumstances, the population mean of the auxiliary variate is known in advance. As a result, double sampling is used. A large sample is chosen to estimate the population mean of the auxiliary variate, and then a subsample is drawn from the large sample or independently from the population in double sampling.

Singh (1967) [3] proposed a ratio-cum-product estimator for population mean using the population mean of two auxiliary variates. Singh and Tailor (2005) [6] proposed a ratio-cum-product estimator for the population mean in simple random sampling based on the correlation coefficient between two auxiliary variables. Tailor and Sharma (2013) [4] proposed a coefficient of kurtosis-based ratio-cum-product estimator of population mean. Sharma *et al.* (2014) [5] proposed a generalised product approach for population mean estimation in two-phase sampling. The ratio-cum-product estimator of Singh and Tailor (2005) [6] is examined in double sampling in this study.

Consider a population $U = \{U_1, U_2, U_3, \ldots, U_N\}$ with a finite size N. Let y be the study variable, and x and z be the auxiliary variables, so that x is positively associated with the study variate y and z is negatively correlated with it.

For estimating the population mean $\bar{Y}$ in simple random sampling, Cochran (1940) [1] defined the classical ratio estimator as

$$\hat{\bar{Y}}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right). \tag{1.1}$$

Robson (1957) [2] proposed using a product estimator to estimate the population mean $\bar{Y}$ as

$$\hat{\bar{Y}}_P = \bar{y}\left(\frac{\bar{z}}{\bar{Z}}\right). \tag{1.2}$$

Singh and Tailor (2003) [7] used the correlation coefficient between the study variable and the auxiliary variate as well as recommended ratio and product type estimators for the population mean $\bar{Y}$ in their analysis as

$$\hat{\bar{Y}}_{STR} = \bar{y}\left(\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}}\right), \tag{1.3}$$

And

$$\hat{\bar{Y}}_{STP} = \bar{y}\left(\frac{\bar{z}+\rho_{yz}}{\bar{Z}+\rho_{yz}}\right), \tag{1.4}$$

where $\bar{y}=\frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{z}=\frac{1}{n}\sum_{i=1}^{n} z_i$ are are unbiased population mean estimators $\bar{Y}, \bar{X}$ and $\bar{Z}$ respectively.

$\hat{\bar{Y}}_R, \hat{\bar{Y}}_P, \hat{\bar{Y}}_{STR}$

and

$\hat{\bar{Y}}_{STP}$ assume that the population mean of the auxiliary variable are known. Double sampling is employed when the population mean of the auxiliary variable is unknown. In a double sampling process,

i. A large sample of size n' is selected to estimate population means of auxiliary variates x and z then
ii. A sample is drawn either as a sub-sample of large sample (case-I) or directly from population independently (case-II).

In "double sampling" classical "ratio and product estimators" are defined as

$$\hat{\bar{Y}}_R^d=\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right), \quad (1.5)$$

and

$$\hat{\bar{Y}}_P^d=\bar{y}\left(\frac{\bar{z}}{\bar{z}'}\right). \quad (1.6)$$

In double sampling, Singh and Tailor (2003) [7] define ratio and product estimators as

$$\hat{\bar{Y}}_{STR}^d=\bar{y}\left(\frac{\bar{x}'+\rho_{yx}}{\bar{x}+\rho_{yx}}\right), \quad (1.7)$$

and

$$\hat{\bar{Y}}_{STP}^d=\bar{y}\left(\frac{\bar{z}+\rho_{yz}}{\bar{z}'+\rho_{yz}}\right). \quad (1.8)$$

Singh (1967) [3] proposed a population mean ratio-cum-product estimator based on the population mean $\bar{Y}$ of two auxiliary variables as

$$\hat{\bar{Y}}_{RP}=\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{z}}{\bar{Z}}\right). \quad (1.9)$$

Singh's (1967) [3] ratio-cum-product estimator is defined as follows in double sampling:

$$.\hat{\bar{Y}}_{RP}^d=\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{z}}{\bar{z}'}\right). \quad (1.10)$$

Suggested Ratio-Cum-Product Estimator

Assume the population mean of two auxiliary variables is $\bar{X}$and $\bar{Z}$ and coefficient of correlation between two auxiliary variables ρ_{xz} are known, Singh and Tailor (2005) [6] proposed a population mean $\bar{Y}$ estimator based on a ratio-cum-product estimator as

$$\hat{\bar{Y}}_{RP}^*=\bar{y}\left(\frac{\bar{X}+\rho_{xz}}{\bar{x}+\rho_{xz}}\right)\left(\frac{\bar{z}+\rho_{xz}}{\bar{Z}+\rho_{xz}}\right), \quad (2.1)$$

where ρ_{xz} is the coefficient of correlation between the auxiliary variables $x$and z.
In many situations, in sequence on population mean of auxiliary variates $\bar{X}$ and $\bar{Z}$ may not available. Singh and Tailor (2005) [6] describe the estimate for this sort of scenario in double sampling as

$$\hat{\bar{Y}}_{RP}^{*d}=\bar{y}\left(\frac{\bar{x}'+\rho_{xz}}{\bar{x}+\rho_{xz}}\right)\left(\frac{\bar{z}+\rho_{xz}}{\bar{z}'+\rho_{xz}}\right). \quad (2.2)$$

For cleanness, it is assumed that the population size N is large in comparisons to sample sizes n and n'. Thus finite population correction term $\left(1-\frac{n}{N}\right)$ and $\left(1-\frac{n'}{N}\right)$ are ignored.

Bias and mean squared error are calculated to compare the proposed estimator to the considered estimators. We write to get the bias and mean squared error of the recommended estimator.
$\bar{y}=\bar{Y}(1+e_0)$,$\bar{x}=\bar{X}(1+e_1)$,$\bar{x}'=\bar{X}(1+e_1')$,$\bar{z}=\bar{Z}(1+e_2)$ and $\bar{z}'=\bar{Z}(1+e_2')$ such that

$$E(e_0)=E(e_1)=E(e_1')=E(e_2)=E(e_2')=0,$$

$E(e_0^2)=\frac{1}{n}C_y^2$, $E(e_1^2)=\frac{1}{n}C_x^2$,

$E(e_1'^2)=\frac{1}{n'}C_x^2$, $E(e_2^2)=\frac{1}{n}C_z^2$,

$E(e'^2_2)=\frac{1}{n'}C_z^2$, $E(e_0e_1)=\frac{1}{n}\rho_{yx}C_yC_x$,

$E(e_0e_1')=\frac{1}{n'}\rho_{yx}C_yC_x$, $E(e_0e_2)=\frac{1}{n}\rho_{yz}C_yC_z$,

$E(e_0e_2')=\frac{1}{n'}\rho_{yz}C_yC_z$, $E(e_1e_1')=\frac{1}{n'}C_x^2$,

$E(e_1e_2)=\frac{1}{n}\rho_{xz}C_zC_x$, $E(e_1e_2')=\frac{1}{n'}\rho_{xz}C_xC_z$,

and

$E(e_2e_2')=\frac{1}{n'}C_z^2$.

Now suggested estimator $\hat{\hat{Y}}_{RP}^{*d}$ can be expressed in terms of e_i's as

$$\hat{\hat{Y}}_{RP}^{*d}=\bar{Y}(1+e_0)\left(\frac{\bar{X}(1+e_1')+\rho_{xz}}{\bar{X}(1+e_1)+\rho_{xz}}\right)\left(\frac{\bar{Z}(1+e_2)+\rho_{xz}}{\bar{Z}(1+e_2')+\rho_{xz}}\right)$$

$$=\bar{Y}(1+e_0)\left(\frac{\bar{X}+\bar{X}e_1'+\rho_{xz}}{\bar{X}+\bar{X}e_1+\rho_{xz}}\right)\left(\frac{\bar{Z}+\bar{Z}e_2+\rho_{xz}}{\bar{Z}+\bar{Z}e_2'+\rho_{xz}}\right)$$

$$=\bar{Y}(1+e_0)\left(\frac{1+\lambda_1e_1'}{1+\lambda_1e_1}\right)\left(\frac{1+\lambda_2e_2}{1+\lambda_2e_2'}\right)$$

$$=\bar{Y}(1+e_0)(1+\lambda_1e_1')(1+\lambda_1e_1)^{-1}(1+\lambda_2e_2)(1+\lambda_2e_2')^{-1}$$

$$=\bar{Y}(1+e_0)(1+\lambda_1e_1')(1-\lambda_1e_1+\lambda_1^2e_1^2)(1+\lambda_2e_2)(1-\lambda_2e_2'+\lambda_2^2e_2'^2)$$

$$=\bar{Y}(1+e_0)(1-\lambda_1e_1+\lambda_1^2e_1^2-\lambda_1^2e_1e_1'+\lambda_1e_1')(1-\lambda_2e_2'+\lambda_2^2e_2'^2+\lambda_2e_2-\lambda_2^2e_2e_2')$$

$$=\bar{Y}(1+e_0)(1-\lambda_1e_1+\lambda_1e_1'+\lambda_1^2e_1^2-\lambda_1^2e_1e_1'-\lambda_2e_2'+\lambda_2^2e_2'^2$$

$$+\lambda_2e_2-\lambda_2^2e_2e_2'+\lambda_1\lambda_2e_1e_2'-\lambda_1\lambda_2e_1e_2\quad)$$

$$=\bar{Y}(1-\lambda_1e_1+\lambda_1e_1'+\lambda_1^2e_1^2-\lambda_1^2e_1e_1'-\lambda_2e_2'+\lambda_2^2e_2'^2+\lambda_2e_2-\lambda_2^2e_2e_2'+\lambda_1\lambda_2e_1e_2'-\lambda_1\lambda_2e_1e_2$$

$$+e_0-\lambda_1e_1e_0-\lambda_2e_0e_2'+\lambda_1e_0e_1'+\lambda_2e_0e_2\quad). \tag{2.3}$$

Taking expectation of both sides of (2.3), we have got

$$E\left(\hat{\hat{Y}}_{RP}^{*d}-\bar{Y}\right)=\bar{Y}E[\lambda_1^2(e_1^2-e_1e_1')+\lambda_2^2(e_2'^2-e_2e_2')+\lambda_1(e_0e_1'-e_0e_1)$$

$$+\lambda_2(e_0e_2-e_0e_2')+\lambda_1\lambda_2(e_1e_2'-e_1e_2)\quad],$$

$$B\left(\hat{\hat{Y}}_{RP}^{*d}\right)=\lambda_1^2\left(\frac{C_x^2}{n}-\frac{C_x^2}{n'}\right)+\lambda_2^2\left(\frac{C_z^2}{n}-\frac{C_z^2}{n'}\right)+\lambda_1\left(\frac{1}{n'}-\frac{1}{n}\right)\rho_{yx}C_yC_x$$

$$+\lambda_2\left(\frac{1}{n}-\frac{1}{n'}\right)\rho_{yz}C_yC_z+\lambda_1\lambda_2\left(\frac{1}{n'}-\frac{1}{n}\right)\rho_{xz}C_xC_z.$$

Finally, the bias of the suggested estimator $\hat{\hat{Y}}_{RP}^{*d}$ upto the first degree of approximation is obtained as

$$B\left(\hat{\hat{Y}}_{RP}^{*d}\right)=\left(\frac{1}{n}-\frac{1}{n'}\right)[\lambda_1^2C_x^2+\lambda_2^2C_z^2-\lambda_1\rho_{yx}C_yC_x+\lambda_2\rho_{yz}C_yC_z-\lambda_1\lambda_2\rho_{xz}C_xC_z\quad] \tag{2.4}$$

Taking the expectation of both sides of (2.3) and squaring it, we get

$$MSE\left(\hat{\hat{Y}}_{RP}^{*d}\right)=\bar{Y}^2E(e_0-\lambda_1e_1+\lambda_1e_1'-\lambda_2e_2'+\lambda_2e_2)^2$$

$= \bar{Y}^2 \left[\frac{C_y^2}{n} + \lambda_1^2 C_x^2 \left(\frac{1}{n} - \frac{1}{n'}\right) + \lambda_2^2 C_z^2 \left(\frac{1}{n} - \frac{1}{n'}\right) - 2\lambda_1 \left(\frac{1}{n} - \frac{1}{n'}\right) \rho_{yx} C_y C_x\right.$

$\left. +2\lambda_2 \left(\frac{1}{n} - \frac{1}{n'}\right) \rho_{yz} C_y C_z - 2\lambda_1^2 \frac{C_x^2}{n'} - 2\lambda_2^2 \frac{C_z^2}{n'} + \frac{2\lambda_1\lambda_2\rho_{xz}C_xC_z}{n'} - \frac{2\lambda_1\lambda_2\rho_{xz}C_xC_z}{n} + \frac{2\lambda_1\lambda_2\rho_{xz}C_xC_z}{n'} + \frac{2\lambda_1\lambda_2\rho_{xz}C_xC_z}{n'}\right] = \bar{Y}^2 \left[\frac{C_y^2}{n} + \left(\frac{1}{n} - \frac{1}{n'}\right) \{\lambda_1^2 C_x^2 + \lambda_2^2 C_z^2 - 2\lambda_1 \rho_{yx} C_y C_x + 2\lambda_2 \rho_{yz} C_y C_z - 2\lambda_1\lambda_2 \rho_{xz} C_x C_z\}\right].$

Finally, the recommended estimator's $\hat{\hat{Y}}_{RP}^{*d}$ mean squared error (MSE) was calculated upto the first degree of approximation can be expressed in case-I as

$$MSE\left(\hat{\hat{Y}}_{RP}^{*d}\right)_I = \bar{Y}^2 \left[\frac{C_y^2}{n} + \left(\frac{1}{n} - \frac{1}{n'}\right) \{\lambda_1 C_x^2 (\lambda_1 - 2K_{01})\right.$$

$$\left. +\lambda_2 C_z^2 (\lambda_2 + 2(K_{02} - \lambda_1 K_{12})) \quad \}\right], \tag{2.5}$$

where

$\lambda_1 = \frac{\bar{X}}{\bar{X} + \rho_{xz}}$ and $\lambda_2 = \frac{\bar{Z}}{\bar{Z} + \rho_{xz}}$.

In case–II suggested estimator $\hat{\hat{Y}}_{RP}^{*d}$ can be expressed as

$$MSE\left(\hat{\hat{Y}}_{RP}^{*d}\right)_{II} = \bar{Y}^2 \left[\left(\frac{1}{n}\right) C_y^2 + \lambda_1 C_x^2 \left\{\lambda_1 \left(\frac{1}{n} + \frac{1}{n'}\right) - \frac{2K_{01}}{n}\right\}\right.$$

$$\left. +C_z^2 \lambda_2 \left\{\lambda_2 \left(\frac{1}{n} + \frac{1}{n'}\right) + \frac{2K_{02}}{n} - \frac{2\lambda_1 K_{12}}{n'}\right\}\right]. \tag{2.6}$$

Case-I: Efficiency Comparisons

In this part, we discuss the circumstances in which the recommended ratio-cum-product type estimator would have lower mean squared error (MSE) than the other estimators discussed.

It is generally known that when using simple random sampling without replacement (SRSWOR), the variance of the sample mean $\bar{y}$ is written as

$$V(\bar{y}) = \bar{Y}^2 \frac{C_y^2}{n}. \tag{3.1}$$

Mean squared error (MSE) of the $\hat{\hat{Y}}_R^d, \hat{\hat{Y}}_P^d, \hat{\hat{Y}}_{STR}^d, \hat{\hat{Y}}_{STP}^d$ and $\hat{\hat{Y}}_{RP}^d$ in Cases I and II are expressed as

$$MSE\left(\hat{\hat{Y}}_R^d\right)_I = \bar{Y}^2 \left[\left(\frac{1}{n}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) C_x^2 (1 - 2K_{01})\right], \tag{3.2}$$

$$MSE\left(\hat{\hat{Y}}_P^d\right)_I = \bar{Y}^2 \left[\left(\frac{1}{n}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) C_z^2 (1 + 2K_{02})\right], \tag{3.3}$$

$$MSE\left(\hat{\hat{Y}}_{STR}^d\right)_I = \bar{Y}^2 \left[\left(\frac{1}{n}\right) C_y^2 + t_1 \left(\frac{1}{n} - \frac{1}{n'}\right) C_x^2 (t_1 - 2K_{01})\right], \tag{3.4}$$

$$MSE\left(\hat{\hat{Y}}_{STP}^d\right)_I = \bar{Y}^2 \left[\left(\frac{1}{n}\right) C_y^2 + t_2 \left(\frac{1}{n} - \frac{1}{n'}\right) C_z^2 (t_2 + 2K_{02})\right], \tag{3.5}$$

$$MSE\left(\hat{\hat{Y}}_{RP}^d\right)_I = \bar{Y}^2 \left[\left(\frac{1}{n}\right) C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) \{C_x^2 (1 - 2K_{01}) + C_z^2 (1 + 2K_{02} - 2K_{12})\}\right], \tag{3.6}$$

The recommended ratio-cum-product estimator $\hat{\hat{Y}}_{RP}^{*d}$ in Case–I would be more efficient than (2.5), (3.1), (3.2), (3.3), (3.4), (3.5), and (3.6), according to comparisons of (2.5), (3.1), (3.2), (3.3), (3.4), (3.5), and (3.6).

i. $\bar{y}$ if

ii. $\frac{C_z^2}{C_x^2} < \frac{\lambda_1 (2K_{01} - \lambda_1)}{\lambda_2 (\lambda_2 + 2(K_{02} - \lambda_1 K_{12}))}$, (3.7)

iii. $\hat{\hat{Y}}_R^d$ if

iv. $\frac{C_z^2}{C_x^2} < \frac{(1 - 2K_{01}) - \lambda_1 (\lambda_1 - 2K_{01})}{\lambda_2 (\lambda_2 + 2(K_{02} - \lambda_1 K_{12}))}$, (3.8)

v. $\hat{\hat{Y}}_P^d$ if

vi. $\frac{C_z^2}{C_x^2} < \frac{\lambda_1 (2K_{01} - \lambda_1)}{\lambda_2 \{\lambda_2 + 2(K_{02} - \lambda_1 K_{12})\} - (1 + 2K_{02})}$, (3.9)

vii. $\hat{\hat{Y}}_{STR}^d$ if

viii. $\frac{C_z^2}{C_x^2} < \frac{t_1(t_1-2K_{01})-\lambda_1(\lambda_1-2K_{01})}{\lambda_2(\lambda_2+2(K_{02}-\lambda_1 K_{12}))}$, (3.10)

ix. $\hat{\bar{Y}}_{STP}^{d}$ if

x. $\frac{C_z^2}{C_x^2} < \frac{\lambda_1(2K_{01}-\lambda_1)}{\lambda_2\{\lambda_2+2(K_{02}-\lambda_1 K_{12})\}-t_2(t_2+2K_{02})}$, (3.11)

xi. $\hat{\bar{Y}}_{RP}^{d}$ if

xii. $\frac{C_z^2}{C_x^2} < \frac{(1-2K_{01})-\lambda_1(\lambda_1-2K_{01})}{\lambda_2\{\lambda_2+2(K_{02}-\lambda_1 K_{12})\}-(1+2(K_{02}-K_{12}))}$, (3.12)

Case-II: Efficiency Comparisons

In Case II, $\hat{\bar{Y}}_{R}^{d}$, $\hat{\bar{Y}}_{P}^{d}$, $\hat{\bar{Y}}_{STR}^{d}$, $\hat{\bar{Y}}_{STP}^{d}$ and $\hat{\bar{Y}}_{RP}^{d}$ are expressed as

$$MSE\left(\hat{\bar{Y}}_{R}^{d}\right)_{II} = \bar{Y}^2\left[\left(\frac{1}{n}\right)C_y^2 + C_x^2\left\{\left(\frac{1}{n}+\frac{1}{n'}\right)-\frac{2K_{01}}{n}\right\}\right], \quad (4.1)$$

$$MSE\left(\hat{\bar{Y}}_{P}^{d}\right)_{II} = \bar{Y}^2\left[\left(\frac{1}{n}\right)C_y^2 + C_z^2\left\{\left(\frac{1}{n}+\frac{1}{n'}\right)+\frac{2K_{02}}{n}\right\}\right], \quad (4.2)$$

$$MSE\left(\hat{\bar{Y}}_{STR}^{d}\right)_{II} = \bar{Y}^2\left[\left(\frac{1}{n}\right)C_y^2 + t_1 C_x^2\left\{t_1\left(\frac{1}{n}+\frac{1}{n'}\right)-\frac{2K_{01}}{n}\right\}\right], \quad (4.3)$$

$$MSE\left(\hat{\bar{Y}}_{STP}^{d}\right)_{II} = \bar{Y}^2\left[\left(\frac{1}{n}\right)C_y^2 + t_2 C_z^2\left\{t_2\left(\frac{1}{n}+\frac{1}{n'}\right)+\frac{2K_{02}}{n}\right\}\right], \quad (4.4)$$

$$MSE\left(\hat{\bar{Y}}_{RP}^{d}\right)_{II} = \bar{Y}^2\left[\left(\frac{1}{n}\right)C_y^2 + C_x^2\left\{\left(\frac{1}{n}+\frac{1}{n'}\right)-\frac{2K_{01}}{n}\right\}\right.$$

$$\left.+C_z^2\left\{\left(\frac{1}{n}+\frac{1}{n'}\right)+\frac{2K_{02}}{n}-\frac{2K_{12}}{n'}\right\}\right] \quad (4.5)$$

Comparing (2.6), (3.1), (4.1), (4.2), (4.3), (4.4), and (4.5), it is clear that the recommended ratio-cum-product estimator $\hat{\bar{Y}}_{RP}^{*d}$ in Case II is more efficient then

i. $\bar{y}$ if

ii. $\frac{C_z^2}{C_x^2} < \frac{\lambda_1\left\{\frac{2K_{01}}{n}-\lambda_1\left(\frac{1}{n}+\frac{1}{n'}\right)\right\}}{\lambda_2\left\{\lambda_2\left(\frac{1}{n}+\frac{1}{n'}\right)+\frac{2K_{02}}{n}-\frac{2\lambda_1 K_{12}}{n'}\right\}}$, (4.6)

iii. $\hat{\bar{Y}}_{R}^{d}$ if

iv. $\frac{C_z^2}{C_x^2} < \frac{(\lambda_1-1)\left\{\frac{2K_{01}}{n}-(\lambda_1+1)\left(\frac{1}{n}+\frac{1}{n'}\right)\right\}}{\lambda_2\left\{\lambda_2\left(\frac{1}{n}+\frac{1}{n'}\right)+\frac{2K_{02}}{n}-\frac{2\lambda_1 K_{12}}{n'}\right\}}$, (4.7)

v. $\hat{\bar{Y}}_{P}^{d}$ if

vi. $\frac{C_z^2}{C_x^2} < \frac{\lambda_1\left\{\frac{2K_{01}}{n}-\lambda_1\left(\frac{1}{n}+\frac{1}{n'}\right)\right\}}{(\lambda_2-1)\left\{(\lambda_2+1)\left(\frac{1}{n}+\frac{1}{n'}\right)+\frac{2K_{02}}{n}\right\}-\frac{2\lambda_1\lambda_2 K_{12}}{n'}}$, (4.8)

vii. $\hat{\bar{Y}}_{STR}^{d}$ if

viii. $\frac{C_z^2}{C_x^2} < \frac{(\lambda_1-t_1)\left\{\frac{2K_{01}}{n}-(\lambda_1+t_1)\left(\frac{1}{n}+\frac{1}{n'}\right)\right\}}{\lambda_2\left\{\lambda_2\left(\frac{1}{n}+\frac{1}{n'}\right)+\frac{2K_{02}}{n}-\frac{2\lambda_1 K_{12}}{n'}\right\}}$, (4.9)

ix. $\hat{\bar{Y}}_{STP}^{d}$ if

x. $\frac{C_z^2}{C_x^2} < \frac{\lambda_1\left\{\frac{2K_{01}}{n}-\lambda_1\left(\frac{1}{n}+\frac{1}{n'}\right)\right\}}{(\lambda_2-t_2)\left\{(\lambda_2+t_2)\left(\frac{1}{n}+\frac{1}{n'}\right)+\frac{2K_{02}}{n}\right\}-\frac{2\lambda_1\lambda_2 K_{12}}{n'}}$, (4.10)

xi. $\hat{\bar{Y}}_{RP}^{d}$ if

xii. $\frac{C_z^2}{C_x^2} < \frac{(\lambda_1-1)\left\{\frac{2K_{01}}{n}-(\lambda_1+1)\left(\frac{1}{n}+\frac{1}{n'}\right)\right\}}{(\lambda_2-1)\left\{(\lambda_2+1)\left(\frac{1}{n}+\frac{1}{n'}\right)+\frac{2K_{02}}{n}\right\}-\frac{2\lambda_1\lambda_2 K_{12}}{n'}}$ (4.11)

Empirical Investigation

A natural population data set is used to examine the percent relative efficiencies (PREs) of different estimators of $\bar{Y}$. The population is explained as follows:

Population [Stockton and Torrie, 1960, p. 282]

y : “Log of leaf burn in sec.”,

x_1 : “Potassium percentage”,

x_2: “Clorine percentage”.

The required population parameters are

$$\bar{Y} = 0.6860,\ C_y = 0.4803,\ \rho_{yx} = 0.1794,\ N = 30,\ n' = 20\ n = 6,$$

$$\bar{X} = 4.6537,\ C_x = 0.2295,\ \rho_{yz} = -0.4996,\ \bar{Z} = 0.8077,\ C_z = 0.7493,\ \rho_{xz} = 0.4074.$$

Table 1: The different estimators' percent relative efficiencies (PREs) with regard to $\bar{y}$

Estimator	PREs
$\bar{y}$	100.00
$\hat{\bar{Y}}_R^d$	96.17
$\hat{\bar{Y}}_P^d$	62.01
$\hat{\bar{Y}}_{STR}^d$	96.84
$\hat{\bar{Y}}_{STP}^d$	65.41
$\hat{\bar{Y}}_{RP}^d$	81.49
$\hat{\bar{Y}}_{RP}^{*d}$	126.21

In comparison to all other examined estimators, Table 5.1 indicates that the recommended ratio-cum-product estimator $\hat{\bar{Y}}_{RP}^{*d}$ has the highest percent relative efficiency (PREs). As a result, when information on the correlation coefficient between the auxiliary variates is available, the proposed ratio-cum-product estimator may be used to estimate the population mean when the requirements in 4.3 and 4.4 are met.

Conclusion

In double sampling, sections 3 and 4 compare the recommended estimate's mean squared errors (MSEs) with the variance of the simple mean estimator, the mean squared errors (MSEs) of the classical ratio and product estimators, Singh and Tailor (2003) [7] estimators, and Singh (1967) [3] estimator. The conditions under which the recommended ratio (4.7) to (4.12) is valid are expressed as (4.7) to (4.12). Estimator of cum-product $\hat{\bar{Y}}_{RP}^{*d}$ has less mean squared error (MSEs) in comparison to $\bar{y}$, $\hat{\bar{Y}}_R^d, \hat{\bar{Y}}_P^d, \hat{\bar{Y}}_{STR}^d, \hat{\bar{Y}}_{STP}^d$ and $\hat{\bar{Y}}_{RP}^d$ in case –I. Similarly, expressions (4.6) to (4.11) are the conditions under which suggested ratio-cum-product estimator $\hat{\bar{Y}}_{RP}^{*d}$ has less mean squared error (MSE) in comparisons to $\bar{y}$, $\hat{\bar{Y}}_R^d, \hat{\bar{Y}}_P^d, \hat{\bar{Y}}_{STR}^d, \hat{\bar{Y}}_{STP}^d$ and $\hat{\bar{Y}}_{RP}^d$ in case –II.

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