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New neoteric ranked set sampling based variant of ranked set sampling

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Abstract
In this paper a new ranked set sampling scheme based of neoteric ranked set named as Neo-Centric ranked set sampling (NCRSS) is proposed for estimating the population mean. The mean under NCRSS were compared with its counterparts under SRS, RSS and DRSS and the results showed that the proposed estimators outperformed its counterparts. It was found that NCRSS produced unbiased and consistent estimators of population mean. A simulation study was conducted to compare the relative efficiency of NCRSS using 1000 simulations for normal, uniform, logistic and Poisson distribution. The results from both empirical and simulation study revealed that the proposed scheme outperforms its counterparts and efficiency is the increasing function of set size m.

Keywords: neo-centric ranked set sampling, RSS, NRSS, CRSS, estimator, simulation and relative efficiency

Introduction
Ranked Set Sampling (*RSS*) technique was first introduced by Mclntyre (1952) [10]. Takahasi and Wakimoto (1968) [12] have provided the necessary mathematical theory of RSS and showed that the sample mean under RSS is an unbiased estimator of the finite population mean and more precise than the sample mean estimator under SRS. Various RSS procedures have been proposed and developed to come up with more efficient estimators for the finite population, (see Al-Saleha and Al-Kadiri (2000) [4], Al-Saleh and Al-Omari (2002) [5], Muttlak (2003) [11], Al-Nasser (2007) [1], Al-Omari *et al.*, (2011) [3], Al-Omari (2011) [2], Haq *et al.*, (2014) [8], Haq *et al.*, (2016), and Zamanzade and Al-Omari (2016) [13]. The ranked set sampling schemes can be described as follows;

1. Identify m^2 units from the target population. Randomly allocate these units into m sets, each of size m.
2. The units within each set are ranked visually or by any inexpensive method with respect to the variable of interest.
3. From the first set of m units, the smallest ranked unit is measured; the second smallest ranked unit is measured from the second set of m units.
4. The process continues until the m^{th} smallest ranked unit is measured from the last set.
5. The process can be repeated r number of times to obtain a large sample of size mr.

The RSS mean estimator and its variance are given by

$$\bar{X}_{RSS} = \frac{1}{rm}\sum_{j=1}^{r}\sum_{i=1}^{m} X_{(i)ij}$$

$$V(\bar{X}_{RSS}) = \frac{\sigma^2}{n} - \frac{1}{rm^2}\sum_{i=1}^{n}(\mu_{(i)} - \mu)^2)$$

$$where\, \mu_{(i)} = \frac{1}{r}\sum_{j=1}^{r} x_{i(i)j}$$

Neoteric Ranked Set Sampling (NRSS)

The NRSS scheme can be summarized as follows:

1. Select a simple random sample of size m^2 units from the target population.
2. Rank the m^2 selected units in an increasing magnitude based on a concomitant variable, personal judgment or any inexpensive method.
3. If k is an odd, then select the $\left[\frac{m+1}{2}+(i-1)m\right]$ th ranked unit for $i=1,\cdots,m$. But if k is an even, then select the $[l+(i-1)m]$ th ranked unit, where $l=\frac{m}{2}$ if i is an even and $l=\frac{m+2}{2}$ if i is an odd for $i=1,\cdots,m$.
4. Repeat Steps 1 through 3 r times (cycles) if needed to obtain a neoteric ranked set sample of size $n = mr$

The estimator of the population mean under NRSS is defined as

$$\bar{Y}_{NRSS}=\frac{1}{mr}\sum_{j=1}^{r}\sum_{i=1}^{m}Y_{[(i-1)m+l]j}$$

With

$$Var(\bar{Y}_{NRSS})=\frac{1}{m^2}\sum_{i=1}^{m}Var\left(Y_{[(i-1)m+l]j}\right)+\frac{2}{m^2}\sum_{i<j}^{m}Cov\left(Y_{[(i-1)m+l},Y_{[(j-1)m+l}\right)$$

Centralized Ranked Set Sampling (CRSS)

Khan *et al.* (2019) [14] introduced new sampling scheme called centralized ranked set sampling (CRSS) for estimating the population mean. Let $Y_{1j},Y_{2j},Y_{3j},\ldots Y_{m^2j}$ be a simple random sample of size m^2 units selected from target population. The CRSS scheme can be described as follows:

1. Select m^2 sample units randomly from the population.
2. The m^2 selected units are then ranked in an increasing order of magnitude based on concomitant variable.
3. Select the $\left(\frac{m^2-m+2i}{2}\right)th$ ranked unit for $i = 1, 2, \ldots . m$
4. Repeat steps 1 to 3 r times to a sample of size $n = mr$

The proposed estimator of the population mean μ under SRSS is given by

$$\bar{Y}_{CRSS}=\frac{1}{mr}\sum_{j=1}^{r}\sum_{i=1}^{m}Y_{\frac{(m^2-m+2i)j}{j}}$$

With,

$$Var(\bar{Y}_{CRSS})=\frac{1}{m^2r}\sum_{i=1}^{m}Var(Y_{\frac{(m^2-m+2i)}{2}}+\frac{2}{m^2}\sum_{i<j}^{m-1}Cov\left(Y_{\frac{m^2-m+2i)}{2}},Y_{\frac{(m^2-m+2j)}{2}}\right)$$

Neo-Centric Ranked Set Sampling (NCRSS)

A new variant of ranked set sampling based on neoteric ranked set is proposed and named as “Neo-Centric Ranked Set Sampling (NCRSS)”. The method is a new modification of ranked set sampling and is suggested for population mean estimation of both symmetric and asymmetric distributions. The said sampling scheme differs from ranked set sampling as all the units are ranked together in both the stages. A step by step description of new scheme is given as:

Description of Neo-Centric Ranked Set Sampling (NCRSS)

Let $Y_{1j},Y_{2j},Y_{3j},\ldots Y_{m^3j}$ be a sample of size m^3 units selected at random from target population. Then N*CRSS* procedure can be summarized as follows:

1 Identify m^3 units from the target population.
2 Randomly allocate m^3 units to m sets each of size m^2 units.
3 Now rank the m^2 units within each of these sets in an increasing order of magnitude based on auxiliary variable
4 If m is an odd, then select the $\left[\frac{m+1}{2}+(i-1)m\right]$ th ranked unit for $i=1,\cdots,m$. But if m is an even, then select the $[l+(i-1)m]$ th $l=\frac{m}{2}$ ranked unit, where $l=\frac{m}{2}$ if i is an even and $l=\frac{m+2}{2}$ if i is an odd for $i=1,\ldots,m$. from each set in order to obtain a sample of size m^2.

5 Again rank the m^2 selected unit with respect to concomitant variable.

6 Select the $\left(\frac{m^2 - m + 2i}{2}\right)th$ ranked unit for $i = 1, 2, \ldots, m$. to obtain a neo-centric ranked set sample of size m.

7 This process can be repeated r times to obtain a neo-centric ranked set sample of size n = mr.

Here, m^2 selected units are ranked together in both the stages, and finally we measure m units out of m^3 units whereas in RSS m units in each are ranked separately. Unlike ranked set sampling the selected units in neo-centric ranked set sampling the measured units

$$\left(Y^{[(i-1)k+l]j}_{\frac{(m \;\; -m+2i)j}{2}}\right) i = 1,2,\ldots,m;\; j = 1,2,\ldots,r$$

are dependent and follow distribution of

$$\left(\frac{m^2 - m + 2i}{2}\right)th$$

order statistics of a sample of size m^3

An example to illustrate PSRSS for estimating the population mean is given below:

Example: Suppose we have to select a neo-centric ranked set sample of size 3 we have to select 27 units randomly from the target population and randomly allocate them to 3 sets each of size 9 to get

$$[Y_{11} \quad Y_{21} \quad Y_{31} \quad Y_{41} \quad Y_{51} \quad Y_{61} \quad Y_{71} \quad Y_{81} \quad Y_{91}]$$

$$[Y_{12} \quad Y_{22} \quad Y_{32} \quad Y_{42} \quad Y_{52} \quad Y_{62} \quad Y_{72} \quad Y_{82} \quad Y_{92}]$$

$$[Y_{13} \quad Y_{23} \quad Y_{33} \quad Y_{43} \quad Y_{53} \quad Y_{63} \quad Y_{73} \quad Y_{83} \quad Y_{93}]$$

Now rank the sets with respect to auxiliary variable to obtain

$$[Y_{(1)1} \quad \underline{Y_{(2)1}} \quad Y_{(3)1} \quad Y_{(4)1} \quad \underline{Y_{(5)1}} \quad Y_{(6)1} \quad Y_{(7)1} \quad \underline{Y_{(8)1}} \quad Y_{(9)1}]$$

$$[Y_{(1)2} \quad \underline{Y_{(2)2}} \quad Y_{(3)2} \quad Y_{(4)2} \quad \underline{Y_{(5)2}} \quad Y_{(6)2} \quad Y_{(7)2} \quad \underline{Y_{(8)2}} \quad Y_{(9)2}]$$

$$[Y_{(1)3} \quad \underline{Y_{(2)3}} \quad Y_{(3)3} \quad Y_{(4)3} \quad \underline{Y_{(5)3}} \quad Y_{(6)3} \quad Y_{(7)3} \quad \underline{Y_{(8)3}} \quad Y_{(9)3}]$$

Now apply neoteric ranked set sampling to obtain

$$[Y_{(2)1} \quad Y_{(5)1} \quad Y_{(8)1} \quad Y_{(2)2} \quad Y_{(5)2} \quad Y_{(8)2} \quad Y_{(2)3} \quad Y_{(5)3} \quad Y_{(8)3}]$$

Again rank the selected units all together to obtain

$$[Y^{(2)1}_{(1)1} \quad Y^{(2)2}_{(2)1} \quad Y^{(2)3}_{(3)1} \quad \underline{Y^{(5)1}_{(4)1}} \quad \underline{Y^{(5)2}_{(5)1}} \quad \underline{Y^{(5)3}_{(6)1}} \quad Y^{(8)1}_{(7)1} \quad Y^{(8)2}_{(8)1} \quad Y^{(8)3}_{(9)1}]$$

Now apply neoteric ranked set sampling to obtain

$$Y^{(5)1}_{(4)1},\; Y^{(5)2}_{(5)1}\, and Y^{(5)3}_{(6)1}$$

which is a neo-centric ranked set sample of size 3.

Estimation Of Population Mean Under NCRSS

The proposed estimator $\bar{Y}_{NCRSS}$ of the population mean μ under NCRSS is given as

$$\bar{Y}_{NCRSS} = \frac{1}{mr} \sum_{j=1}^{r} \sum_{i=1}^{m} Y^{[(i-1)k+l]j}_{\frac{(m \;\; -m+2i)j}{2}}$$

This estimator is unbiased only if the underlying distribution is symmetric. The associated variance is given by

$$Var(\bar{Y}_{NCRSS}) = \frac{1}{m^2 r}\left[\sum_{i=1}^{m} Var\left(Y^{[(i-1)k+l]}_{\left(\frac{m^2-m+2i}{2}\right)}\right) + 2\sum_{i<j}^{m} Cov\left(Y^{[(i-1)k+l]}_{\left(\frac{m^2-m+2i}{2}\right)}, Y^{[(j-1)k+l]}_{\left(\frac{m^2-m+2j}{2}\right)}\right)\right]$$

Lemma: $\bar{Y}_{NCRSS}$ is an unbiased estimator of population mean if the ranking is perfect and the parent distribution is symmetric. Proof: for r=1, the NCRSS estimator of population mean μ, can be written as

$$\bar{Y}_{NCRSS} = \frac{1}{m}\sum_{i=1}^{m} Y^{[((i+1)-1)m+l]}_{\frac{(m \quad -m+2i)}{2}}$$

$$\bar{Y}_{NCRSS} = \frac{1}{m}\left[Y^{2m+l}_{\frac{(m^2-m+2)}{2}} + Y^{3m+l}_{\frac{(m^2-m+4)}{2}} + Y^{3m+l}_{\frac{(m^2-m+6)}{2}} + \ldots + Y^{m^2-1+l}_{\frac{(m^2-m+2m-2)}{2}} + Y^{m^2+l}_{\frac{(m^2-m+2m)}{2}}\right]$$

Taking expectation we have

$$E(\bar{Y}_{NCRSS}) = \frac{1}{m}\left[E\left(Y^{2m+l}_{\frac{(m^2-m+2)}{2}}\right) + E\left(Y^{3m+l}_{\frac{(m^2-m+4)}{2}}\right) + E\left(Y^{4m+l}_{\frac{(m^2-m+6)}{2}}\right) + \cdots + E\left(Y^{m^2-1+l}_{\frac{(m^2-m+2m-2)}{2}}\right) + E\left(Y^{m^2+l}_{\frac{(m^2-m+2m)}{2}}\right)\right] =$$
$$\frac{1}{m}\left[\mu^{2m+l}_{\frac{(m^2-m+2)}{2}} + \mu^{3m+l}_{\frac{(m^2-m+4)}{2}} + \mu^{4m+l}_{\frac{(m^2-m+6)}{2}} + \cdots + \mu^{m^2-1+l}_{\frac{(m^2-m+2m-2)}{2}} + \mu^{m^2+l}_{\frac{(m^2-m+2m)}{2}}\right]$$

For symmetric distribution, we have $\mu_{(i)} - \mu = \mu - \mu_{(n-i+1)}$, (David and Nagaraja, 2003) [6]. Thus

$$\mu^{2m+l}_{\frac{m^2-m+2}{2}} - \mu = \mu - \mu^{m^2+l}_{\frac{m^2-m+2m}{2}} \quad and \quad \mu^{3m+l}_{\frac{m^2-m+4}{2}} - \mu = \mu - \mu^{m^2-1+l}_{\frac{m^2-m+2m-2}{2}}$$

Also $E\left(Y^{\frac{m^2+1}{2}}_{\frac{m^2+1}{2}}\right) = \mu^{\frac{m^2+1}{2}}_{\frac{m^2+1}{2}} = \mu$ since it is the median of selected m^2 units.

When m is odd we have

$$E\left(\bar{Y}_{NCRSS(0)}\right) = \frac{1}{m}\left(\frac{m-1}{2}(2\mu) + \mu\right) = \mu$$

Whereas, if m is even then

$$E\left(\bar{Y}_{NCRSS(E)}\right) = \frac{1}{m}\left(\frac{m}{2}(2\mu) + \mu\right) = \mu$$

This shows that $\bar{Y}_{NCRSS}$ is an unbiased estimator of population mean μ is NCRSS Scheme.

Application of NCRSS

To outline the NCRSS scheme, the data on yield of HDP of Gala Redlum apple variety during the year 2016 maintained at plate I high density plantation at SKUAST-Kashmir is used.. The descriptive statistics of the variables used in the study is presented in table-1.

Table 1: Descriptive Statistics of Yield, Height and TCA of Gala Redlum.

Descriptive statistics	Yield	TCA
Mean	10.22	6.299
Standard Error	0.196	0.065
Median	10.293	6.127
Confidence Level (95.0%)	0.388	0.128
Standard Deviation	2.280	0.753
Sample Variance	5.199	0.566
Kurtosis	0.536	0.344
Skewness	0.559	0.856
Range	12.972	3.701
Minimum	5.697	5.083
Maximum	18.669	5.783

Suppose we have to draw a neo-centric ranked set sample of size *m*=3. Then the procedure of NCRSS is shown below:

Step-1: We have to draw a random sample of 27 plants. These plants are randomly allocated to 3 sets each of size 9. These plants are represented (using ordered pair of TCA and Yield) as:

Set 1: [(5.64, 7.94), (6.72, 11.77), (6.92, 12.0), (5.08, 5.70), (6.26, 10.50), (6.58, 11.43), (5.71, 8.20), (6.44, 11.21), (8.32, 16.22)]

Set 2: [(5.99, 9.44), (6.47, 11.23), (6.64, 11.64), (5.86, 9.21), (5.86, 9.21), (6.64, 11.64), (5.57, 7.68), (6.13, 10.29), (7.24, 12.55)]

Set 3: [(5.64, 7.94), (6.26, 10.50), (6.58, 11.43), (5.08, 5.70), (5.71, 8.20), (5.80, 8.92), (5.84, 9.04), (6.47, 11.23), (6.64, 11.64)]
Step-2: Now rank the pairs within each set in an increasing order of magnitude based on TCA (Shown in bold) as

Set 1: [(5.08, 5.70), (5.64, 7.94), (5.71, 8.20), (6.26, 10.50), (6.44, 11.21), (6.58, 11.43), (6.72, 11.77), (6.92, 12.0), (8.32, 16.22)]

Set 2: [(5.57, 7.68), (5.86, 9.21), (5.86, 9.21), (5.99, 9.44), (6.13, 10.29), (6.47, 11.23), (6.64, 11.64), (6.64, 11.64), (7.24, 12.55)]

Set 3: [(5.08, 5.70), (5.64, 7.94), (5.71, 8.20), (5.80, 8.92), (5.84, 9.04), (6.26, 10.50), (6.47, 11.23), (6.58, 11.43), (6.64, 11.64)]

Step-3: Now select the units with rank 2, 5, 8 to obtain a neoteric ranked set sample of size 9 as
[(5.64, 7.94), (6.44, 11.21), (6.92, 12.0), (5.86, 9.21), (6.13, 10.29), (6.64, 11.64), (5.64, 7.94), (5.84, 9.04), (6.58, 11.43)]

Step-4: Again rank the pairs selected in step-3 in a ascending order of magnitude based on TCA (shown as bold) as
[(5.64, 7.94), (5.64, 7.94), (5.84, 9.04), (5.86, 9.21), (6.13, 10.29), (6.44, 11.21), (6.58, 11.43), (6.64, 11.64), (6.92, 12.0)]

Step-5: Now select the units with rank 4, 5, 6 to obtain a neo-centric ranked set sample of size m=3.
(5.86, 9.21), (6.13, 10.29), (6.44, 11.21)

Therefore 9.21, 10.29, 11.21 are selected for estimating the population mean yield of Gala Redlum.
Using the equation below to estimate population mean.

$$\bar{Y}_{NCRSS} = \frac{1}{mr}\sum_{j=1}^{r}\sum_{i=1}^{m} Y_{\frac{(m\ \ -m+2i)j}{2}}^{[(i-1)k+l]j}$$

$$\bar{Y}_{NCRSS} = \frac{1}{3}(9.21 + 10.29 + 11.21) = 10.24$$

Empirical Study
In this section, an empirical study on real data set is performed to investigate the performance of NCRSS in estimating the population mean. The data on the yield, of Gala Redlum is considered as the variable of interest and TCA as the auxiliary variable. When the underlying distribution is symmetric, the efficiency of NCRSS relative to SRS, RSS and DRSS is given by:

$$eff(\bar{Y}_{NCRSS}, \bar{Y}_{SRS}) = \frac{Var(\bar{Y}_{SRS})}{Var(\bar{Y}_{NCRSS})},$$

$$eff(\bar{Y}_{NCRSS}, \bar{X}_{RSS}) = \frac{Var(\bar{Y}_{RSS})}{Var(\bar{Y}_{NCRSS})}$$

$$eff(\bar{Y}_{NCRSS}, \bar{Y}_{DRSS}) = \frac{Var(\bar{Y}_{DRSS})}{Var(\bar{Y}_{NCRSS})}$$

When the underlying distribution is asymmetric, the efficiency of NCRSS relative to SRS, RSS and DRSS is given by:

$$eff(\bar{Y}_{NCRSS}, \bar{Y}_{SRS}) = \frac{MSE(\bar{Y}_{SRS})}{MSE(\bar{Y}_{NCRSS})},$$

$$eff(\bar{Y}_{NCRSS}, \bar{Y}_{RSS}) = \frac{MSE(\bar{Y}_{RSS})}{MSE(\bar{Y}_{NCRSS})},$$

$$eff(\bar{Y}_{NCRSS}, \bar{Y}_{DRSS}) = \frac{MSE(\bar{Y}_{DRSS})}{MSE(\bar{Y}_{NCRSS})}$$

Where,

$$MSE(\bar{Y}) = Var(\bar{Y}) + [Bias(\bar{Y})]^2$$

Summary Results for estimating the population mean of yield of HDP
To compare the performance of estimators of Population mean under NCRSS, the mean and the corresponding MSE's for estimators were calculated using various sampling schemes and the results are presented in Table. 2, Table-3 and Table-4.

Table 2: Efficiency of NCRSS over SRS

Size	Simple Random Sampling		Neo-Centric Ranked Set Sampling		Efficiency
	MEAN	MSE	MEAN	MSE	
m=3	10.92	2.587	9.65	1.391	1.86
m=4	9.58	2.413	10.25	0.331	7.29
m=5	10.36	1.764	9.91	0.178	9.91
m=6	10.21	1.336	10.08	0.141	9.48
m=7	10.35	1.022	10.34	0.093	10.99
m=8	10.40	0.988	10.17	0.075	13.17

Table 3: Efficiency of NCRSS over RSS

Size	Ranked Set Sampling		Neo-Centric Ranked Set Sampling		Efficiency
	MEAN	MSE	MEAN	MSE	
m=3	10.64	1.797	9.65	1.391	1.29
m=4	10.39	1.337	10.25	0.331	4.04
m=5	9.75	1.041	9.91	0.178	5.85
m=6	10.29	0.650	10.08	0.141	4.61
m=7	10.16	0.471	10.34	0.093	5.06
m=8	10.16	0.452	10.17	0.075	6.03

Table 4: Efficiency of NCRSS over DRSS

Size	Double Ranked Set Sampling		Neo-Centric Ranked Set Sampling		Efficiency
	MEAN	MSE	MEAN	MSE	
m=3	9.56	1.798	9.65	1.391	1.29
m=4	10.22	0.815	10.25	0.331	2.46
m=5	10.00	0.694	9.91	0.178	3.90
m=6	10.05	0.586	10.08	0.141	4.16
m=7	10.11	0.416	10.34	0.093	4.47
m=8	10.06	0.402	10.17	0.075	5.36

Table 5: Relative Efficiency of NCRSS over SRS, RSS and DRSS for different distribution.

Distribution	m	$V(\bar{Y}_{SRS})$	$V(\bar{Y}_{RSS})$	$V(\bar{Y}_{DRSS})$	$V(\bar{Y}_{NCRSS})$	$RE_1(\bar{Y}_{SRS},\bar{Y}_{NCRSS})$	$RE_2(\bar{Y}_{RSS},\bar{Y}_{NCRSS})$	$RE_3(\bar{Y}_{DRSS},\bar{Y}_{NCRSS})$
Normal (0,1)	3	1.114	0.847	0.703	0.552	2.02	1.53	1.27
	4	0.814	0.590	0.551	0.126	6.46	4.68	4.37
	5	0.706	0.450	0.377	0.090	7.84	5.00	4.19
	6	0.223	0.066	0.077	0.013	17.15	5.08	5.92
Uniform (0,1)	3	0.092	0.080	0.060	0.039	2.36	2.05	1.54
	4	0.065	0.055	0.051	0.028	2.32	1.96	1.82
	5	0.045	0.040	0.025	0.011	4.09	3.64	2.27
	6	0.023	0.008	0.007	0.004	5.75	2.0	1.75
Logistic (0,1)	3	0.514	0.257	0.218	0.097	5.30	2.65	2.25
	4	0.467	0.209	0.151	0.067	6.97	3.12	2.25
	5	0.352	0.070	0.038	0.019	18.53	3.68	2.00
	6	0.326	0.164	0.039	0.028	11.64	5.86	1.39
Poisson (10)	3	2.333	1.815	0.778	0.667	3.50	2.72	1.17
	4	1.667	1.292	0.604	0.422	3.95	3.06	1.43
	5	1.200	0.832	0.512	0.368	3.26	2.26	1.39
	6	1.600	0.711	0.466	0.208	7.69	3.42	2.24

The results reveal that NCRSS dominates SRS, RSS and DRSS even though the data is not symmetric since the coefficient of skewness is not zero as depicted by the descriptive statistics table 1 and produces unbiased and consistent estimators of the population mean of the yield of high density plants. It is found that MSE of NCRSS estimators is relatively smaller than MSE of estimators under SRS, RSS and DRSS. The smaller MSE of NCRSS than SRS, RSS and DRSS is due to the fact that NCRSS excludes the effect of the outliers / extremes observations in both the stages and concentrates only on the middle fraction of the selected units. The MSE of NCRSS estimators was found to be smaller when set size *m* is 8 (0.075) and was highest when set size *m* is 3 (1.391). As the sample size increases, MSE under NCRSS approaches to 0 indicating that the proposed estimators are consistent. NCRSS helps in reducing the effect of extreme cum outlier observations because the extreme observations are not selected in the sample. While computing estimates of the population mean under NCRSS, as *m* increases, the resulting sequence of estimates converges in probability to true population mean as the scheme itself concentrates at the centre of the selected units leaving out extreme observations. This means that the distributions of the estimates become increasingly concentrated near the true value of the parameter being estimated so that the probability of the estimator being arbitrarily close to the true population.

Simulation Study

A simulation study is carried out to compare the performances of estimators. Estimates of variances of mean estimators were

computed under SRS, RSS, DRSS and NCRSS taking *m*=3, 4, 5, 6 from normal, uniform, logistic and poisson distribution using 1000 simulations. Estimators are compared interms of relative efficiencies (RE) and the results are presented in the Table 5**.**
The results indicate that NCRSS dominates in all the cases. The results show that relative efficiency is an increasing function of *m*. From the simulation study it is found that the estimators under NCRSS have relatively lower variance as compared to its counterparts as is concentrates on the central part of the selected units in case of all the distributions.

Conclusion

NCRSS sampling strategy can prove very useful sampling studies where exact quantification of a selected unit is either difficult or costly in terms of time, money or labor, but the units can be ranked with a reasonable success on the basis of the auxiliary variables which are more accessible and also correlated with the variable of interest as well as in the situations where outliers are present in the data. The suggested estimator increases the efficiency of estimating population mean. The empirical and simulation study showed that the suggested that the is superior that its counter parts under SRS, RSS and DRSS

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