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Construction of polygonal designs for sampling from circularly ordered populations

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Abstract
Balanced sampling plans excluding adjacent units or BSA (α) plans are useful for sampling from populations in which the nearer units provide similar observations due to natural ordering of the units in time or space. The ordering of units in the population may be circular. For BSA (α) plans, all the first order inclusion probabilities are equal whereas second order inclusion probabilities for pairs of adjacent units at a distance less than or equal to α are zero and constant for all other pairs of non-adjacent units which are at a distance greater than α. An important series of incomplete block designs called polygonal designs that are suitable for obtaining BSA (α) plans. Considering the blocks of polygonal designs as samples and the treatments as units, a BSA (α) plan can be obtained by assigning an equal probability of selection to the blocks. Computer algorithm which is based on linear integer programming approach available for generating efficient binary incomplete block designs. This algorithm has been improved for obtaining polygonal designs for the circular ordered structure of population units. In this study, we obtain new balanced sampling plans excluding adjacent units for one dimensional population with the circular ordering of units.

Keywords: Balanced sampling plans, linear programming approach, polygonal designs

1. Introduction

In any field of scientific investigation, data need to be collected as per the objectives of the study. Process of data collection may involve laboratory experiments, field trials, sample surveys etc. Sample survey gains importance in the context when a huge population needs to be studied for its characteristics under the constraints of cost and/or time. Sampling is concerned with the selection of a subset of individuals from a population to estimate parameters of interest. The basic objectives of the sampling theory are the development of sample selection procedure and estimation method to obtain the precise estimates of population parameters. Simple random sampling is a basic selection procedure, which provides an equal chance of selection to all possible samples in the sample space. There do occur many situations where providing the equal probability of selection to all possible samples is not a very desirable feature and controls may be desirable for selection procedures which may provide the basis of the preferability of the samples.

There exists some positive correlation between the neighbouring units when the population units are in the natural ordering in time or space. In such a situation, neighbouring units in the population may provide a similar response for the variable under study. While taking the sample from such populations, it is intuitively logical to avoid adjacent units in the sample. In such situations where the neighbouring units provide a similar response, Balanced Sampling plans Excluding Contiguous units (BSEC plans) can be used. BSEC plans, developed by (Hedayat *et al*., 1988) [4], are those sampling plans in which the pairs of contiguous units never appear in a sample whereas all other pairs appear equally often in the samples. Further, Stufken (1993) [14] developed BSA (α) plans by generalizing the concept of BSEC plans, i.e., excluding all those pairs of units whose distance is less than or equal to α. In BSA(α) plans, all the first order inclusion probabilities are equal whereas second order inclusion probabilities for pairs of adjacent units at a distance less than or equal to α are zero and constant for all other pairs of non-adjacent units which are at a distance greater than α. Here, first order inclusion probability (π_i) is the probability that a unit i, i = 1, 2, …, N, will be included in the sample and

second order inclusion probability (π_{ij}) is the probability that a pair of units (i, j), $i \neq j$= 1, 2, …, N, will be included together in a sample, where N denote the population size.
In BSA (α) plans usually a circular ordering of units in the population is assumed, i.e., if there are N units in the population, units 1 and N are considered adjacent. Circular ordering is assumed for convenience and simplicity. The first and second order inclusion probabilities of BSA (α) plan for drawing a sample of size n from a population of size N with a circular ordering of units are given by $\pi_i = \frac{n}{N}$, i = 1, 2,…, N and $\pi_{ij} = \frac{n(n-1)}{N(N-2\alpha-1)}$, $i \neq j$ = 1,2,…, N if $\delta(i, j) > \alpha$ and π_{ij} = 0 if $\delta(i, j) \leq \alpha$, where $\delta(i, j)$ is the distance between 2 units i and j. Here, N is the population size and n is the sample size. These inclusion probabilities are valid for circular structures only. Polygonal Designs (PDs) are presented by (Stufken *et al.* 1999) [16] and showed that PD's are equivalent to BSA plans. A polygonal design is an arrangement of N symbols in b blocks of size n with r replications and distance α such that

1. Any two symbols i, j with distance less than or equal to α do not appear together in a block.
2. Any other pair of symbols i, j with a distance greater than α appear together in precisely λ blocks.

In this case, the parameters of the design are N, b, r, n, λ and α. The parameter of the design satisfies the following conditions.

- $Nr = bn$
- $\lambda(N - 2\alpha - 1) = r(n - 1)$ (1)

If α = 0, a polygonal design reduces to Balanced Incomplete Block (BIB) design. Let a sample of size n is taken from a population of size N and n treatments in each block be considered as the units in the sample, of the b blocks constitute the sample space S, then selecting one block randomly with the probability of selection p(s) =1/b ∀ s = 1, 2, …, b is equivalent to BSA(α) plan. Using the parametric relations of polygonal designs, the first order inclusion probability will be, $\pi_i = \frac{r}{b}$, i = 1, 2, …, N and second order inclusion probability will be, $\pi_{ij} = \frac{\lambda}{b}$, $\forall i \neq j$ = 1, 2, …, N, for distance between units greater than α and $\pi_{ij} = 0$, $\forall i \neq j$ = 1, 2, …, N, for distance between units less than or equal to α.
Most of the works on BSA plans assume one dimensional population though there are some works on two dimensional BSA plans, e.g., (Bryant *et al.* 2002; Wright 2008; Gopinath *et al.* 2017) [1, 17, 13]. In this article, we control ourselves to one dimensional population. BSEC plans were obtained under the assumption of circular ordering of units. Under one dimensional circular ordering, the distance between two units i and j is denoted by $\delta(i, j) = Min\{|i-j|, N-|i-j|\}$.
We shall denote a balanced sampling plan excluding adjacent unit for population size N, sample size n and distance α in general as BSA (N, n, α). A BSA (N, n, α) under circular is represented as cBSA (N, n, α).
For given parameters N, n and α, there is a lot of interest in the existence and construction of polygonal designs. A large number of polygonal designs are acquired by several authors (Hedayat *et al.* 1988; Colbourn and Ling 1999; Stufken *et al.* 1999; Stufken and Wright 2008; Mandal *et al.* 2008; Mandal *et al.* 2011; Tahir *et al.* 2012; Gupta *et al.* 2012; Mandal *et al.* 2014; Kumar *et al.* 2019) [4, 17, 2, 10, 3], but still there are gaps in these design parameters. Therefore, extra efforts are required to gain polygonal designs for given combinations of N, b and n. A linear programming approach is described by (Wright and Stufken 2008; Mandal *et al.* 2008) [17] to obtain smaller cBSAs which can also be applied to find more cBSAs. In this work, we obtain new cBSA with smaller support sizes using an algorithm developed by (Kumar et al. 2016). This algorithm can construct cyclic as well as non-cyclic cBSAs.

2. Materials and methods

We are going to describe the methodology for the construction of polygonal designs through the computer-aided search for the circular ordering of population units. The polygonal designs have a one-to-one correspondence with balanced sampling plans excluding adjacent units up to a distance of $\alpha \geq 1$ (BSA (α) plans). Given a polygonal design with N symbols in b blocks each of size n such that every pair of symbols which are at a distance more than α occur together in λ blocks, selecting a block with probability 1/b gives a sample of BSA (α) plan. Linear programming approach is exploited for obtaining linear BSA (α) plans.
An incidence matrix $\mathbf{N} = (n_{is})$ of a block, the design is a rectangular array with N rows and b columns with entries n_{is}, where n_{is} represents the number of times i^{th} treatment appears in s^{th} block. If all n_{is} are either 0 or 1 then the corresponding block design is binary. The elements of incidence matrices of complete block designs are all positive ($n_{is} \geq 1$). Incomplete block designs have incidence matrices with at least one element equal to zero. The matrix $\mathbf{NN'}$is termed as the concurrence matrix of the design and it is the properties of this matrix which determine the desirability of a particular design. Concurrence matrix has certain informative properties, particularly interesting for binary design. For any binary design, it can be seen that the i^{th} diagonal element of $\mathbf{NN'}$is equal to r_i, the number of blocks in which the i^{th}treatment occurs, and the off-diagonal element in the i^{th}row, and the i'^{th} column is equal to the number of blocks in which the i^{th} and i'^{th} treatment concur. Hence, the off-diagonal elements are usually called concurrence and denoted sometimes by $\lambda_{ii'}$.
Under circular ordering, the concurrence matrix of the polygonal designs is known. For given α, the distance between adjacent units, r, replication of each treatment and λ, the number of blocks in which pair of treatment at a distance greater than α occurs together. The algorithm attempts to obtain the incidence matrix $\mathbf{N}$ of the required polygonal design. First, the user needs to put input N, b, k, λ, α and r for cBSA. In the first step, the first row of the incidence matrix $\mathbf{N}$ is obtained by randomly allotting 1 to r columns (blocks) out of b available columns of the $\mathbf{N}$ matrix. Next row of the $\mathbf{N}$ matrix is obtained in such a way that the desired concurrence of the second row with the first row is achieved and this is done with the help of an integer linear programming formulation. Likewise, the third row is obtained such that desired concurrences of the third row with the first row and the second row are achieved. This process is continued till all N rows are obtained. There may be a chance that at some row, no solution is obtained. In that case, one of the previously obtained rows are deleted and an alternative solution to that deleted row is obtained by using an integer linear programming formulation and then the current row is

obtained.

2.1 Algorithm for generation of polygonal designs in case of circular ordering

We modified the linear integer programming approach to obtain the polygonal designs with parameters *N*, *b*, *r*, *n*, λ and α that was introduced by Mandal *et al.* (2014) [10]. Let *b* represents the number of blocks and r_i = number of replications of i^{th} treatment. The steps of the algorithms are described in the sequel:

Step 1: Construct a row vector of order 1 × *b* with 1 in randomly chosen *r* positions and 0 in the remaining *b* - *r* positions of the vector. Denote this 1 × *b* matrix as M [1]. Set $\mathbf{T} = \beta$. The role of **T** will become clear later.

Step *i* (*i* =2, 3, …, *N*): Obtain weights $w_s = \frac{1}{n_s}$ whenever $n_s > 0$ and $w_s = 1$ if $n_s = 0$, Where $n_s = \sum_{i'}^{i-1} p_{i's}$, *s* = 1, 2, …, *b* is the size of the s^{th} block from (*i* – 1) × b matrix $M^{(i-1)}$, and where $p_{i's}$ is the element at the i'^{th} row and the s^{th} column (i' = 1, 2,...,*i* -1; *s* = *1, 2,...,b*) of $\mathbf{M}^{(i-1)}$. Then, to obtain the row *i* = 2, 3, …, *N* of the incidence matrix **N.** We solve the following linear integer programming problem for the row *i* concerning binary decision variables $x_1, x_2, \ldots, x_b$:

$$\text{Maximize} \quad \varphi = \sum_{s=1}^{b} w_s r_s$$

Subject to constraints

$$\text{i)} \quad \sum_{s=1}^{b} x_s = r_i$$

$$\text{ii)} \quad x_s \le n - n_s \forall s = 1, 2, \ldots, b \qquad (2)$$

$$\text{iii)} \quad \left.\begin{aligned} \sum_{s=1}^{b} n_{i's} x_s &= \lambda \text{ if } (i',i) \text{ non-adjacent} \\ &= 0 \text{ if } (i',i) \text{ adjacent} \end{aligned}\right\} i' = 1, 2, \ldots, i-1$$

The above linear programming problem is such that there exists an optimal solution at Step *i*, or there is no optimal solution at Step *i*.

Step *i*a: If there exists an optimal solution at the i^{th} step (*i* = 2, 3, ..., *N*), then set $\mathbf{M}^{(i)} = \begin{pmatrix} \mathbf{M}^{(i-1)} \\ \mathbf{x}'_N \end{pmatrix}$ where (x'_N= (x_{1N}, x_{2N}, …,x_{bN}) denotes an optimal solution at the i^{th} step. Obtain weights w_s, s = 1, 2, …, b as before. Solve the formulation for the next *i*.

Step *i*b: If there is no optimal solution at the Step *i*, select a random number *m* between 1 to (*i* -1), set $\mathbf{T} = \begin{pmatrix} \mathbf{T} \\ \mathbf{n}'^{(i-1)}_m \end{pmatrix}$ where $n'^{(i-1)}_m$ denote the m^{th} row of the $\mathbf{M}^{(i-1)}$ matrix, and then set $n'^{(i-1)}_m = 0'$. We then try to obtain an alternative solution for the m^{th} row of the incidence matrix. For this, we solve the following LIP formulation:

$$\text{Maximize } \phi = \sum_{s=1}^{b} w_s r_s$$

Subject to constraints

$$\text{i)} \quad \sum_{s=1}^{b} x_s = r_m$$

$$\text{ii)} \quad x_s \le n - n_s \forall s = 1, 2, \ldots, b \qquad (3)$$

$$\text{iii)} \quad \sum_{j=1}^{b} n_{i's} x_s = \begin{cases} \lambda & if\, \delta(m,i') > 0 \\ and & i' = 1, 2, \\ 0 & if\, \delta(m,i') \le \alpha \end{cases}$$

$$\text{iv)} \quad \sum_{s=1}^{b} t_{qs} x_s \le r - 1, \forall q = 1, 2, \ldots, p$$

where t_{qs} indicates the element at the q^{th} row and the s^{th} column of the **T** matrix. The last constraint in the formulation (3) ensures that the deleted rows stored in **T** do not appear again as a solution. An optimal solution to the formulation (3) gives an alternative solution for the m^{th} row of the incidence matrix. If the formulation (3) does not have a feasible solution, we try deleting another row. Once a solution for the m^{th} row is obtained, we continue to obtain the i^{th} row as before using formulation (2). We stop when all the *N* rows of the incidence matrix are obtained. Within parameter range $N \le 60$, $n \le 7$, $\alpha \le 5$, the parameters of the polygonal designs satisfying the parametric relations of polygonal designs with circular ordering of units are obtained. After incorporating the concurrence matrix of Polygonal designs for the circular ordering of units, a package *ibd* available in R software that is applicable for construction of polygonal design for the above parameters.

3. Results

In this unit, we describe the results of polygonal designs for the circular ordering of population units. Polygonal designs for circular ordering have been found in the range *R* = {$N \le 60$, $n \le 7$, $\lambda \le 10$, $\alpha \le 7$}, using the modified algorithm based on linear integer programming approach. We divide the parametric range *R* as $R = R_1 \cup R_2$ where R_1= {$N \le 50$, $n \le 7$, $\lambda \le 7$, $\alpha \le 5$} which is already covered by (Kumar *et al.* 2016, 2019) and R_2 present the remaining parametric range in *R* not covered by them. The parameters of the polygonal designs within parameter range *R* = {$N \le 60$, $n \le 7$, $\alpha \le 7$ satisfying the parametric conditions of polygonal designs with the circular ordering of units are obtained. After incorporating the concurrence matrix of Polygonal designs for the circular ordering of units, for construction of polygonal design for the above parameters where R package *ibd* is applied. Above developed designs are compared with existing designs available in (Hedayat *et al.* 1988; Stufken 1993; See *et al.* 1997; Stufken *et al.* 1999; Colbourn and Ling 1998; Mandal *et al.* 2008; Tahir *et al.* 2012) [4, 16, 15, 2, 8].

In the above given parametric range, a total of 3812 polygonal designs satisfies necessary conditions of existence of polygonal designs. Table No.1 represents the distribution of these 3812 designs for α = 1, 2, 3, 4, 5, 6 and 7. It also represents the new designs obtained through the algorithm, the number of designs for which either the solution is unknown or non-existence is not proved.

Table 1: Polygonal designs for circular ordering in parametric range *R*

	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$	$\alpha = 5$	$\alpha = 6$	$\alpha = 7$	Total
Number of parametric combinations	388	428	427	369	433	971	796	3812
Number of designs obtained	113	103	86	81	80	289	279	1031
Number of designs exists but not obtained	97	104	96	68	56	0	0	421
Number of non-existing designs	38	75	106	127	216	538	452	1552
Number of designs for which solution is unknown	132	137	132	89	81	134	55	760
Number of new designs	8	9	7	4	0	10	10	48

In a given above Table 1, based on changing the value of α parameter, we can identify that 1552 designs are non-existent as per the Theorem numbers 4.3(1) of Stufken *et al.* (2008) [17] and Result 2.1 of Parsad *et al*. (2007) [12] out of total 3812 polygonal designs. There are 760 designs for which their solution is unknown (row 5) and in the third row, we can find that there is also 421 design which exists but not obtained. Out of 1031 designs obtained using the above algorithm, only 48 designs are new whose solution was not presented in the literature earlier. The parameters of these 48 new observed designs are given in below Table 2.

Table 2: Parameters of newly obtained designs under the circular ordering of population units

Sl. No.	*v*	*B*	*R*	*k*	λ	α	Remarks
1	19	152	40	5	10	1	
2	20	136	34	5	8	1	
3	23	138	36	6	9	1	
4	51	408	32	4	2	1	
5	51	816	48	3	2	1	
6	54	459	34	4	2	1	
7	54	918	51	3	2	1	
8	57	513	36	4	2	1	
9	15	225	45	3	9	2	
10	51	391	23	3	1	2	
11	51	782	46	3	2	2	
12	53	484	48	3	2	2	
13	53	424	32	4	2	2	
14	54	882	49	3	2	2	
15	56	476	34	4	2	2	
16	59	531	27	3	1	2	
17	59	531	36	4	2	2	
18	51	748	44	3	2	3	
19	51	1122	66	3	3	3	
20	52	390	30	4	2	3	
21	52	780	45	3	2	3	
22	52	585	45	4	3	3	
23	52	780	60	4	4	3	
24	55	440	32	4	2	3	
25	51	357	21	3	1	4	
26	51	714	42	3	2	4	
27	54	810	45	3	2	4	
28	57	912	48	3	2	4	
29	39	169	13	3	1	6	
30	39	338	26	3	2	6	2 copies of design at Sl. No. 29
31	39	507	39	3	3	6	3 copies of design at Sl. No. 29
32	39	676	52	3	4	6	4 copies of design at Sl. No. 29
33	39	845	65	3	5	6	5 copies of design at Sl. No. 29
34	39	1014	78	3	6	6	6 copies of design at Sl. No. 29
35	39	1183	91	3	7	6	7copies of design at Sl. No. 29
36	39	1352	104	3	8	6	8 copies of design at Sl. No. 29
37	39	1521	117	3	9	6	9 copies of design at Sl. No. 29
38	39	1690	130	3	10	6	10 copies of design at Sl. No. 29
39	45	225	15	3	1	7	
40	45	450	30	3	2	7	2 copies of design at Sl. No. 39
41	45	675	45	3	3	7	3 copies of design at Sl. No. 39
42	45	900	60	3	4	7	4 copies of design at Sl. No. 39
43	45	1125	75	3	5	7	5 copies of design at Sl. No. 39
44	45	1350	90	3	6	7	6 copies of design at Sl. No. 39
45	45	1575	105	3	7	7	7 copies of design at Sl. No. 39
46	45	1800	120	3	8	7	8 copies of design at Sl. No. 39
47	45	2025	135	3	9	7	9 copies of design at Sl. No. 39
48	45	2250	150	3	10	7	10 copies of design at Sl. No. 39

From Table 2, it can be easily observed that out of 48 new designs 18 designs could be obtained by taking copies of other designs. In this study, the polygonal designs have been obtained within parameter range $N \leq 60$, $n \leq 7$, $\alpha \leq 7$ but the above-used algorithm is much generalised and it can also be used for obtaining polygonal designs outside this range. The modified algorithm is also very general and can be used for obtaining polygonal designs outside this parametric range $N \leq 60$, $n \leq 7$, $\lambda \leq 10$, $\alpha \leq 7$ also.

4. Conclusion

The modified algorithm has been applied to obtain polygonal designs in the parametric range $N \leq 60$, $n \leq 7$, $\lambda \leq 10$, $\alpha \leq 7$. 3812 designs fulfil the parametric conditions for the existence of a circular polygonal design. Out of these 3812 circular polygonal designs, 1031 designs have been obtained. It is found that 48 designs are new, which are not available in the literature. In the case of circular ordering of population units, the total number of unknown designs are 760 that need a solution. Thus, further research is needed to obtain the solution of these polygonal design and to prove the non-existence of these designs. The modified algorithm is general and can be used for obtaining polygonal designs outside this parametric range for the circular ordering of units. For large designs, it may, however, be tome prohibitive. Therefore, further research efforts are required for obtaining polygonal designs for large population and sample sizes.

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