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The Pharma Innovation

ISSN (E): 2277- 7695
ISSN (P): 2349-8242
NAAS Rating: 5.03
TPI 2020; 9(7): 234-238

www.thepharmajournal.com
Received: 16-05-2020
Accepted: 20-06-2020

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Stability analysis of milling process using bifurcation theory

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Abstract
Chatter produced during machining process is a constraint to productivity and quality of the machining process and product. Stability lobe diagram is the common tools used to select the optimum combination of speed and depth of cut. Though the boundary between stable and unstable region can be predicted using stability diagram, the stable and unstable behavior cannot be explored with lobe diagrams. In this article milling process is studied for stability using bifurcation theory. This theory refers to the change in behaviour of the cutting tool system with respect to change in control parameter – axial depth of cut. Bifurcation diagram for milling process is constructed from the displacements obtained by numerical integration of force equations using time domain solutions. Stability is predicted based on the presence of diverging profiles of displacements. In addition Poincare maps are constructed through which phase space trajectory can be observed. These maps are subsequently used to study stability behavior including period n Bifurcations. Bifurcation diagram and their results are presented for various stable and unstable conditions of cut and compared with stability lobe diagrams.

Keywords: Bifurcation diagram, regenerative chatter, time domain simulation, poincare plot

Introduction

During machining process machine tool chatter is an inherent phenomenon. It is a self-excited vibration which is produced due to the interaction between work piece and machine tool. Presence of chatter during machining process results into negative effects on cutting tool and workpiece such as poor surface finish of product, faster too wear, excessive noise, waste of material, waste of money, reduced material removal rate etc. so in designing and manufacturing process chatter is considered as one of the most limiting factor. Chatter in machining is produced due to free vibrations, forced vibration and self-excited vibrations. Free vibrations occur due to incorrect tool path which leads to collision between the workpiece and cutting tool. The primary source of forced vibration in milling process is entry and exit of cutting edge during machining process. However these are also produced due to unbalance of rotating members, instable servo system etc. Both free and forced vibrations can be reduced, avoided or eliminated when cause of vibration is known. But self-excited vibrations which is produced by the interaction between workpiece and cutting tool is the most undesirable and least controllable.so study of chatter plays an important role in the field of dynamics.

Tlusty [1] showed that Machine tool chatter is mainly produced as a result of regenerative effect which produces when machine tool removes the surface which was created during previous pass of the tool. Machine-tool chatter explanation was given by Tobias [2], and Merritt [3] as "regeneration of waviness." They represented information regarding stability as a function of the control parameters such as spindle speed and depth of cut by developing stability lobe diagram with loops. From this it is possible identify stable and unstable cutting regions from which maximum chatterless chip width is predicted as a function of spindle speeds. Analytical method presented by Altintas and Budak [4] for predicting stability lobes on high speed milling have derived various equations for constructing stability lobes.

The above mentioned method is widely used to obtain chatter free machining environment, but in recent times in order to obtain deeper insight of chatter alongwith stable and unstable boundry regions, solutions of equation of motion using time domain are studied with numerical integration which are solved in time domain. Time domain simulations are particularly useful in the study of: 1) onset of chatter, 2) post chatter problems, 3) loss of tool contact conditions, 4) multiple regenerative effect 5) effect of both work piece and tool flexibilities and 6) effect of non-linearities such as frictional, thermal and impact chatter. With the advances in the modern theory on non-linear dynamics and latest computer hardware it is possible to analyze the above problems easily.

In addition, time domain simulation methods are also preferred to study low immersion cuts where a combined analysis of flexible tool and work piece system is required [5]. Li Zhongqun, Liu Qiang [6] and Li. H., and Li. X [7] presented model using time domain for the analysis and simulation of milling process. it makes use of several criterias for chatter stability to overcome the limitations for single chatter stability criteria. The cutting forces produced are calculated for instantaneous undeformed chip thickness, properties of working material, cutting conditions and dynamics of milling system.

The recent focus of several researches and invistigations has been the occurrence of bifurcation concept during cutting process. Bifurcation is a change in qualitative character of a solution with respect to change in control parameter [7]. Bifurcation theory is the study of how the solution and their multiplicities changes as their control parameter changes. Tony schimtz [8] described extended milling bifurcation diagram using numerical simulation technique. From this diagram it is possible to study both stability behavior such as period n and hopf bifurcation and amplitude of vibrations produced during stable and unstable conditions. Zhao and Balchandran [9] has implemented time domain simulation method in which loss of contact between tool and workpiece and regenerative effect is studied. From this study they identified secondary hopf bifurcation and stated that for low immersion milling period doubling bifurcation occurs. Davies *et al.* 2000 [10] used frequency signal from microphone to plot both period 2 and hopf instability. They also provided experimental validation for these instabilities for both up milling and down milling.

The objective of this paper is to study the milling process stability using bifurcation theory. Time domain simulation is carried out for different cutting conditions to observe displacement profiles. These displacement profile are studied and data is used to plot bifurcation diagram to classify chatter and no chatter condition as well as behaviour of the preocess during cutting process. In the following section modelling of dynamic milling process is presented and governing equations of chatter are described.

Limitatins of stability lobe diagram and use of bifurcation diagram to overcome such limitations is explained in section 3 & 4. Numerical time domain results and bifurcation diagram is presented in section 5 whie in section 7 conclusions obtained from this study are given.

2. Modeling of dynamic milling process

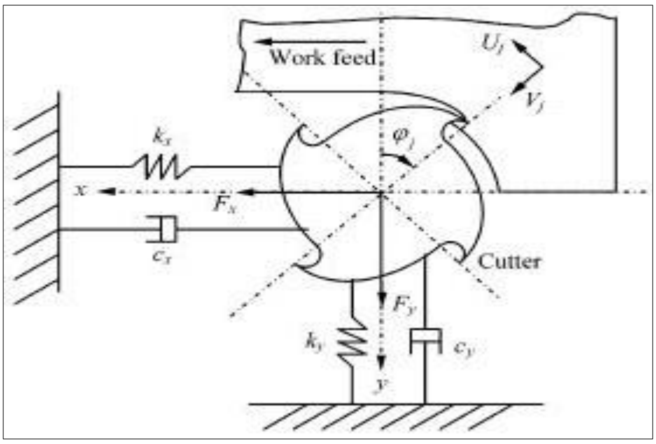

Fig 1: Dynamic model of milling system

An existing instantaneous cutting force model is introduced for the following analysis. Milling tool model is considered to as two DOF vibration system in two orthogonal directions. This model considers self-excited regenerative mechanism as the principle source of chatter.

Equation of motion for the milling process consisting of n number of cutting edges can be described in X and Y direction by following differential equation

$$m_x \ddot{x} + c_x\dot{x} + k_x = \sum_{j=0}^{Z} F_t \cos\theta - F_r \sin\theta \qquad (1)$$

$$m_y \ddot{y} + c_y\dot{y} + k_y = \sum_{j=0}^{Z} -F_t \cos\theta - F_r \sin\theta \qquad (2)$$

Where Z indicates number of teeth on milling cutter, c is the damping coefficient, k is the stiffness of the machine tool, F_t and F_r are the tangential and radial cutting forces acting on a tooth.

Tangential and radial cutting forces can be expressed using cutting coefficient (K_t, K_r) and chip width h by following equations

Tangential force $(F_t) = K_t h(\theta) a$ (3)

Radial force $(F_r) = F_t K_r$ (4)

The cutting tool dynamics of the end mill cutter is influenced by the parameters namely mass, stiffness and damping factor. Equation can be rewritten in global x-y coordinates as

$$\ddot{x} + 2\xi_x\omega_n\dot{x} + \omega_{nx}^2 x = \frac{1}{m_x}\sum_{j=0}^{z} F_{xj} = \frac{1}{m_x}F_x(t) \qquad (5)$$

$$\ddot{y} + 2\xi_x\omega_n\dot{y} + \omega_{nx}^2 y = \frac{1}{m_y}\sum_{j=0}^{z} F_{yj} = \frac{1}{m_y}F_y(t) \qquad (6)$$

Where $\omega_n = \sqrt{\frac{k}{m}}$ is natural angular frequency, $\xi = \frac{c}{2\sqrt{km}}$ is the relative damping factor, F_{xj} and F_{yj} are the component of the cutting forces which is applied to jth cutting tooth in x and y direction respectively. A Z toothed cutter may engage with multiple teeth on the work piece depending on the immersion angle of the cut. Hence the forces on every tooth computed are summed up to consider the effect of all the teeth engaged in the cut as shown in Eqns. 3 & 4.

In milling process both cutter and workpiece vibrates due to cutting forces. The cutting tooth removes the wavy surface produced during previous pass which results into modulation of chip thickness. since change in chip thickness produced is depends on both current pass (t) and previous pass (t- T), it can be expressed in local coordinates as

$$h_d = (h_s + [x(t) - x(t-T)]\, sin(\theta_i)[y(t) - y(t-T)]\, cos(\theta_i)) \qquad (7)$$

Where

$h_s = f_t \sin\theta$ and f_t = feed rate per flute, $\boldsymbol{\theta}$ is the instantaneous angle made by tooth which can be calculated as summation of tool rotation angle and pitch angle of respective tooth

$$\theta(t) = \frac{2\pi NT}{60} + \frac{z2\pi}{N} \qquad (8)$$

T is the tooth passing period and is given by

$$T = \frac{60}{NZ} \qquad (9)$$

Where N is the speed of cutter and Z is the number of tooth

In machining with end-mills, the tool edge does not engage with work all the times. The tool engagement time is dependent on the angle of entry and exit. This produces intermittent cutting operation. To consider the effect of intermittent cutting action of the tooth of the milling cutter a unit step function $g(\theta_i)$ is multiplied with the above equation. It must be noted that at any given point of time, depending on the number of tool edges and entry and exit angle, more than one cutting edge may be involved in the cutting operation.

$$g(\theta) = \begin{cases} 1 & \theta_{entry} < \theta < \theta_{exit} \\ 0 & otherwise \end{cases} \qquad (10)$$

Using Eqn. 8 the instantaneous position of the tool edge can be computed. If it lies between the entry and exit angle of the work, forces can be obtained according to Eqn.1& 2 otherwise forces experienced from that edge will be zero according to Eqn. 10.

3. Stability lobe diagram

The cutting process stability can be determined by the construction of stability lobe diagram. This diagram is constructed for the spindle speed and axial depth of cut. In this diagram border between stable and unstable regions are indicated. Fig. shows border consists of several lobes. So area below the lobe is stable region of cutting process where area above the lobe is unstable region of cutting process.

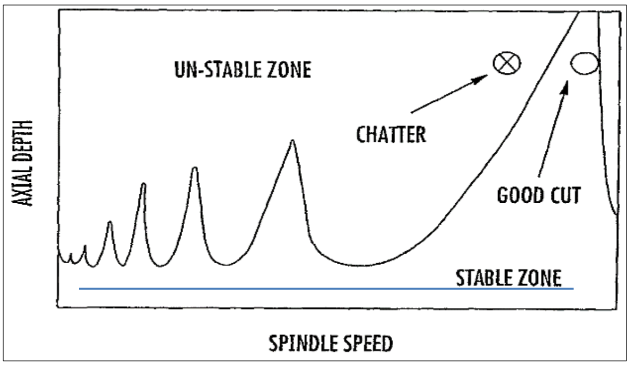

Fig 2: Example of stability lobe diagram

Stability lobe diagram can be devided into three regions *viz.* unconditionally stabl, conditionally stable and unconditionally unstable. In unconditionally stable region process is always stable which is independent of chatter frequency and spindle speed. The lowest points on the lobe can be connected by horizontal line. The area below this line is unconditionally stable. Similarly we can draw a line connecting all intersection points of lobe. The region above this line is unconditionally unstable region. The region between these two horizontal lines is conditionally stable. In this, points above the lobe are unstable whereas stable points are below the lobes.
The use of stability lobe diagram is limited upto to predict only stable and unstable region of cutting process. Unstable behavior of cutting process cannot be predicted in stability lobe diagram. The detail unstable (periodic motion,chatter) behavior can be predicted with bifurcation diagram which is discussed in next section.

4. Bifurcation diagram

Change in behaviour of a system with respect to change in control parameter is called bifurcation. Bifurcation diagram helps to study system behavior like tool motion, amplitude of vibration with respect to control parameters such as axial depth of cut, spindle speed during machining process. For milling process, tool displacement in x and y direction is sampled for a given depth of cut at particular speed in time domain simulation. Displacement values are then collected for a given axial depth of cut at a particular interval of time. The same procedure is followed for different axial depth of cut at constant speed.
As shown in fig. In stable cutting condition amplitude points repeat itself after each revolution because response of vibration is periodic with spindle rotation. So superposition of all these points gives a single point on bifurcation diagram. Further increase in axial depth of cut leads to secondary hopf instability which is also called Neimark sacker bifurcation and then quasi periodic motion occurs. In this case amplitude points do not repeat and distribution of these points appears as vertical line in bifurcation diagram.

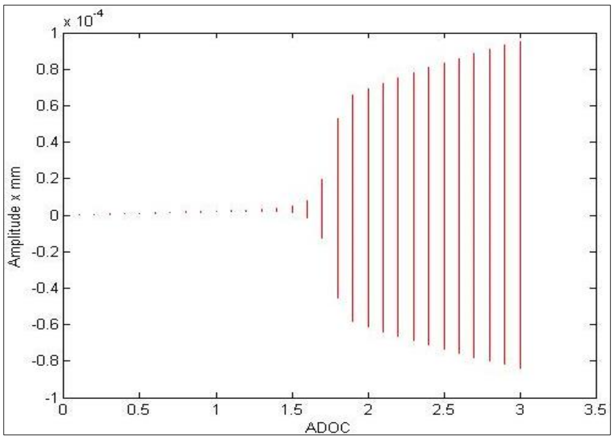

Fig 3: Bifurcation Diagram

5. Results and Discussions

Table 1 shows the data used to evaluate time domain simulation in MATLAB. A straight fluted end milling cutter is used for simplicity. The chatter delay differential equations are solved and the time domain solutions are computed for various speeds and depth of cut.

Table 1: Modal parameter for MATLAB simulation

Natural freq. ω_{nx}, ω_{ny} (Hz)	665,795
Damping ratio ξ_x, ξ_y	0.016, 0.018
Stiffnessk_x k_y (N/m)	7e7, 1e8
Number of teeth	8
Milling type	Half immersion

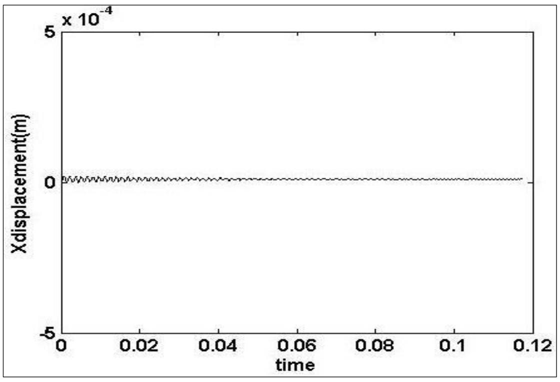

Fig 4: Displacement profile (8000 rpm,1mm ADOC)

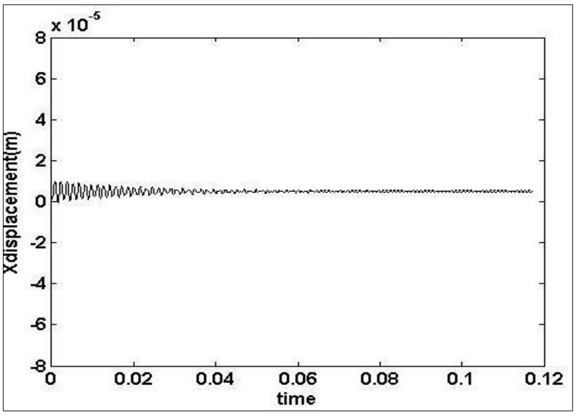

Fig 5: Displacement profile (8000 rpm,1.5mm ADOC)

From this displacements are observed for for various depth of cut with certain step. Figs. 4 & 5,6,7 shows the displacement profile for varios depth of cut at for a constant spindle speed of 8000 rpm. Cutter vibration (Displacements) Converges to zero upto depth of cut of 1.8 mm. it Indicate stable region of cutting process as shown in fig 4 & 5.

But when it crosses this ADOC of 1.8 mm displacement values starts increasing and diverging profile of displacement values is achieved as shown in fig.6 which indicates start of unstable region of cutting process.

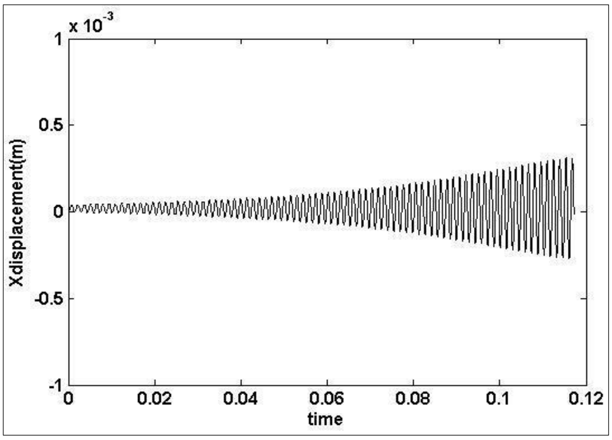

Fig 6: Displacement profile (8000 rpm,1.8mm ADOC)

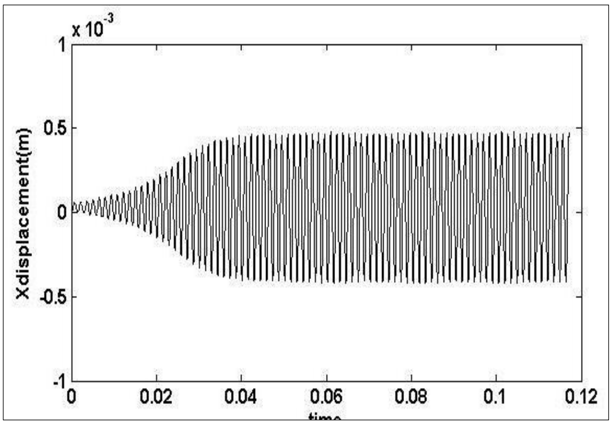

Fig 7: Displacement profile (8000 rpm,2mm ADOC)

So Depth of cut of 1.8 mm is a critical depth of cut for given cutting conditions.It can be also called as Bifurcation Point. Beyond this depth of cut periodic motion occures utpo approximately 8 mm ADOC. During this motion displacement values are repeated for particulat interval of time. Further increase in depth of cut leads to chaotic motion as shown in Fig.8.

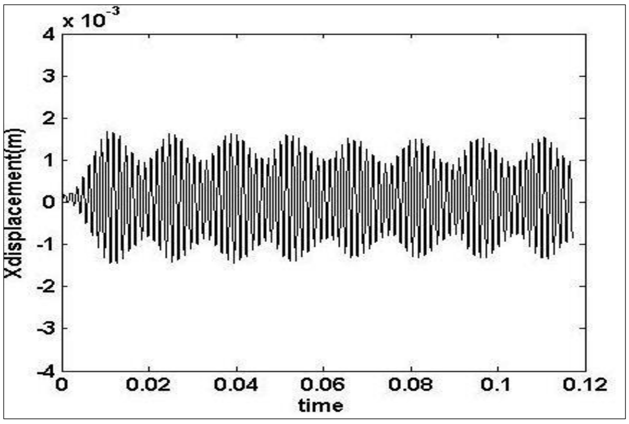

Fig 8.1: Displacement profile (8000 rpm,8 mm ADOC)

Poincare maps can be developed to observe phase space trajectory by plotiing the displacement produced versus velocity. For stable cutting condition motion is periodic with tooth period so sampled points repeat itself and grouping of points is observed as shown in fig. 8

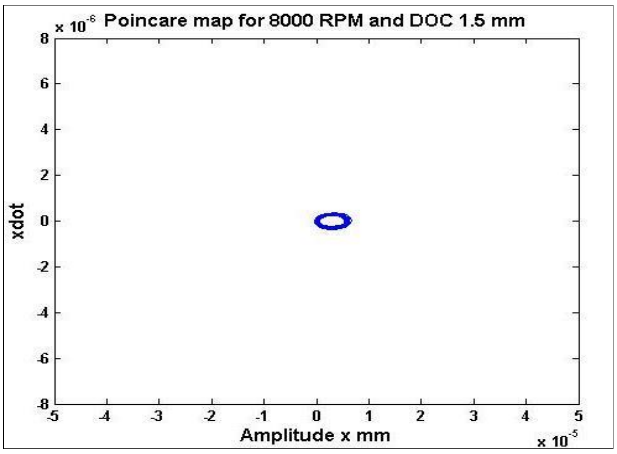

Fig 8.2: Poincare Map (8000 rpm,1.5mm ADOC)

Whereas for unstable cuts sampled points do not repeat and form a elliptical distribution with scatterd points as shoen in fi.9.

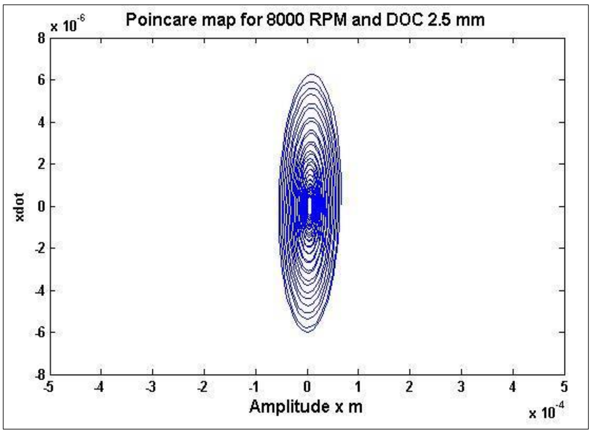

Fig 8.3: Poincare Map (8000 rpm,2.5mm ADOC)

Displacement values are selected for particualr interval of time when process is stabilized for certain state. And these displacemnt values are plotted with respect to depth of cut. fig.8 shows bifurcation diagram for milling process for a spindle speed of 8000 rpm. The process is stable upto an axial depth of cut of 1.8 mm shown by horizontal dotted line. Each dot indicates superposition of all repeated points for particular depth of cut.

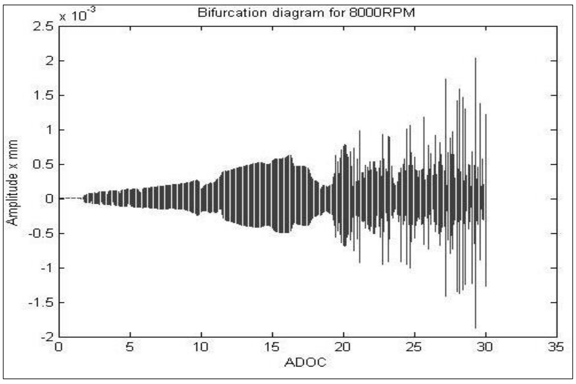

Fig 8.4: Bifurcation Diagram for 6000rpm

Transition between stable to unstable periodic motion occurs at an axial depth of cut of 1.8 mm which can be called as bifurcation point.

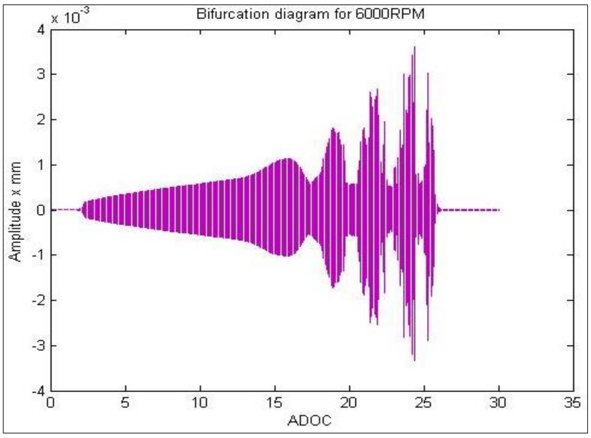

Fig 9: Bifurcation Diagram for 8000 rpm.

A dramatic reduction in displacement or amplitude of vibrarion is observed for some depth of cuts, even though cutting process is remain unstable. For some cutting cinditions, stable process can be achieved even for higher depth of cut as shown in fig. 9. This bifurcation diagram can be drawn for various depth of cut to explore cutting conditions. Fig. 9,10 shows similar bifurcation diagram for a spindle speeds of 6000 and 10000 rpm respectively.

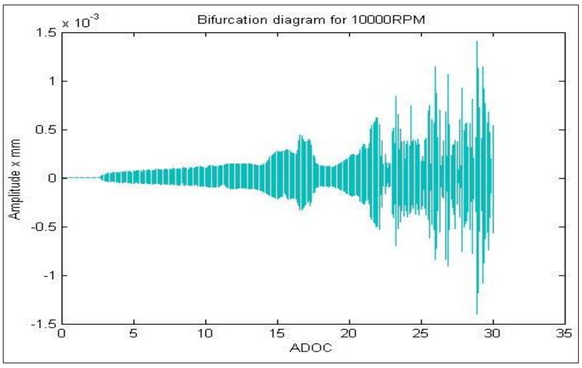

Fig 10: Bifurcation Diagram for 10000 rpm.

From these bifurcation diagrams it is possible to analyse stability condition for various spindle speeds and axial depth of cuts. Table 2 shows critical depth of cut (bifurcation points) for various spindle speeds of machining process. choosing of depth of cut smaleer than critical depth of cut for given spindle speed will give stable machining process.

Table 2: Results obtained from bifurcation Diagram

Speed(rpm)	Critical depth of cut (Bifurcation point)
2000	1.8
4000	3.2
6000	1.9
8000	1.5
10000	2.9

6. Conclusions

In this paper, Bifurcation Diagram for milling process is described using time domain Simulation method. It is possible to understand stability behavior and amplitude of vibrations produced during both stable and unstable cutting conditions. It is possible to know the amplitude of vibration produced for each depth of cut during machining process. A new approach to obtain time domain solution to chatter problems with non-linearity is proposed in this paper. Though a straight flute cutter is used, it can be extended to helical flute cutter by modifying the program segment that computes forces. This proposed method can be used as alternative to stability lobe diagram. It can be extended to solve chatter problems for turning.

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