Effect of partial slip on the pulsatile flow of an ionized gas in an inclined channel

LM Vishnupriya, S Sreenadh and A Ramadevi

Abstract
The influence of partial slip on the pulsatile flow of an ionized gas between two porous plates is studied. The channel is inclined at an angle $\theta$ with the horizontal. The ionized gas is injected into the channel from the lower porous plate with velocity $V$ and is sucked out of the upper porous plate with the same velocity. The analytical solutions are obtained for velocity and mass flux. Influence of various parameters on the flow characteristics have been discussed with the help of graphs. The numerical results obtained reveal many interesting behaviors. It is observed that the velocity decreases with increased Hartmann number $M$ and increases with increasing hall parameter $m$. Reynolds number $R$ and slip parameter $\beta$ in both fully and partially ionized cases.

Keywords: Hall currents, MHD, ionized gas, pulsatile flow, inclined channel

1. Introduction
The importance of the study of the pulsatile flow in a channel or a porous pipe is too well known to be elaborated. It has biological applications in relation to hemodynamics, industrial applications in relation to heat exchange efficiency, applications in natural systems like circulatory systems, respiratory systems, vascular diseases, in engineering systems like reciprocating pumps, IC engines, combusters and applications in MEMS micro fluidic engineering applications. The terms ‘pulsatile’, ‘oscillatory’ or ‘unsteady’ are generally used in the literature to describe the flows in which velocity or pressure or both depend on time. Oscillatory flow is a periodic flow that oscillates around a zero value. Pulsatile flow is a periodic flow that oscillates around a mean value not equal to zero, i.e., it is a steady flow on which is superposed an oscillatory flow.

In medical sciences, for example, dialysis of blood in artificial kidneys, pumping of blood in arteries bounded by endothelium, the fluid in the synovial joint bounded by cartilages are modelled as channels bounded by porous layers. Particularly, the pulsatile flow in a porous channel is important in understanding the process of dialysis of blood in an artificial kidney. These flows may be classified as pulsatile flows and hence such investigations find important applications in biomedical engineering.

or microwave generator. Ionized gas (plasma) is in our more immediate surroundings. It is the stuff of lightning, of computer chip manufacturing, of flat panel televisions, and of lamps that are the light of our lives. One of the important application of ionized gas in pharmacy is to sterilize some medical products. Some studies reveal that alkaline ionized water can be used for the treatment of cancer cells.

The boundary conditions for the flow through porous beds need special attention. Generally the no-slip condition is valid on the boundary when a fluid flows between impermeable surfaces. But when it flows between permeable surfaces, the no-slip condition is no longer valid since there will be a migration of fluid, tangential to the boundary within the permeable surfaces. The velocity within the permeable beds will be different from the velocity of the fluid in the channel and we have to match the two velocities at the interface. Based on their experimental investigations, Beavers and Joseph \[\text{[11]}\] proposed a slip condition at the nominal surface of the permeable bed.

All the various organs present in human beings and animals are not horizontal in general. In order to understand such phenomenon, it will be interesting to study pulsatile flow in an inclined channel bounded by permeable beds. To the extent, the present authors have surveyed the pulsatile flow of an ionized gas between two porous plates in an inclined channel has not been studied so far. In view of several applications in engineering and medicine, the effect of partial slip on the pulsatile flow of an ionized gas in an inclined channel is investigated in this chapter.

1.1 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(u, w)</td>
<td>Velocity components along x and z directions</td>
</tr>
<tr>
<td>(\overline{q})</td>
<td>Velocity vector</td>
</tr>
<tr>
<td>(V)</td>
<td>Suction / injection velocity</td>
</tr>
<tr>
<td>(B)</td>
<td>The magnetic flux density</td>
</tr>
<tr>
<td>(E)</td>
<td>The electric field</td>
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<tr>
<td>(j)</td>
<td>The current density</td>
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<tr>
<td>(p)</td>
<td>Pressure</td>
</tr>
<tr>
<td>(E_x, E_z)</td>
<td>Electric fields along x and z directions</td>
</tr>
<tr>
<td>(s = \frac{p_e}{p})</td>
<td>Ionization parameter (the ratio of the electron pressure to the total pressure)</td>
</tr>
<tr>
<td>(\sigma_0)</td>
<td>The coefficient of proportionality between the current density and collision term in the equation of motion of charged particles</td>
</tr>
<tr>
<td>(\sigma_1, \sigma_2)</td>
<td>The modified conductivities parallel and normal to the direction of electric field</td>
</tr>
<tr>
<td>(u_c = \left(\frac{\partial p}{\partial x}\right) \frac{h^2}{\nu})</td>
<td>Characteristic velocity</td>
</tr>
<tr>
<td>(m = \frac{w_e}{\tau_e})</td>
<td>Hall parameter</td>
</tr>
<tr>
<td>(w_e)</td>
<td>The gyration frequency of electron</td>
</tr>
<tr>
<td>(\tau_e, \tau_i)</td>
<td>The mean collision time between electron, ion and electron, neutral particles respectively</td>
</tr>
<tr>
<td>(M = \sqrt{\frac{B^2 \sigma_0}{\mu \nu}})</td>
<td>Hartmann Number</td>
</tr>
<tr>
<td>(B_0)</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Coefficient of viscosity (\mu = \rho \nu)</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Density of the fluid</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Kinematic Viscosity</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Slip parameter</td>
</tr>
<tr>
<td>(\omega)</td>
<td>Frequency of the flow</td>
</tr>
<tr>
<td>(R = \frac{Vh}{\nu})</td>
<td>Suction Reynolds number</td>
</tr>
</tbody>
</table>

2. Mathematical formulation of the problem

We consider the pulsatile flow of a viscous incompressible ionized gas between two porous plates, extending along x- and z-directions with an angle of inclination \(\theta\) (Fig. 3.1). The fluid is injected into the channel from the lower porous plate with a velocity \(V\) and is sucked out into the upper porous plate with same velocity normal to these walls i.e. along y-direction. The
height of the channel is denoted by \( 2h \) and the width is assumed to be very large in comparison with the channel height \( 2h \). We employ the first order velocity slip conditions at both the walls.

The x-axis is taken in the direction of hydrodynamic pressure gradient in the plane parallel to the channel walls. A parallel uniform magnetic field of strength \( B_0 \) is applied in the y-direction and the hall currents are taken into account. All physical quantities except pressure become functions of y only, as the walls are infinite in extent along x- and z-directions. Further to obtain the governing equations, the following assumptions are made following Sato (1961).

1. The density of gas is everywhere constant.
2. The ionization is in equilibrium which is not affected by the applied electric and magnetic fields.
3. The effect of space charge is neglected.
4. The flow is fully developed, that is
   \[
   \frac{\partial p}{\partial x} = 0 \quad \text{and} \quad \frac{\partial}{\partial z} = 0 \quad \text{except} \quad -\frac{\partial p}{\partial x} \neq 0
   \]
5. The magnetic Reynolds number is small (so that the externally applied magnetic field is undisturbed by the flow). The induced magnetic field is small when compared with the applied field. Therefore, components in the conductivity tensor are expressed in terms of \( B_0 \).
6. The fluid is driven by an unsteady pressure gradient given by
   \[
   \frac{1}{\rho} \frac{\partial p}{\partial x} = A + B e^{\imath \omega t}
   \]
   where \( p \) is the pressure, \( \rho \) is the density, \( t \) is the time, \( A \) and \( B \) are constants and \( \omega \) is the frequency.

The physical configuration and the nature of the flow suggest the following forms of velocity vector \( \mathbf{q} \), the magnetic flux density \( \mathbf{B} \), the electric field \( \mathbf{E} \) and the current density \( \mathbf{J} \):

\[
\mathbf{q} = [u, 0, w], \quad \mathbf{B} = [0, B_y, 0], \quad \mathbf{E} = [E_x, 0, E_y] \quad \text{and} \quad \mathbf{J} = [j_x, 0, j_z]
\]

In view of the above assumptions, the governing equations of motion reduce to

\[
\left( \frac{1}{\rho} \frac{\partial p}{\partial x} \right) \left[ \mathbf{q} \cdot \sigma_u \right] + \frac{\partial^2 u}{\partial y^2} + \frac{B_0}{\rho} \left[ -\sigma_z (E_x + u B_y) + \sigma_z (E_z - w B_y) \right] = V \frac{\partial u}{\partial y} - g \sin \theta
\]
(2)

\[
\left( \frac{1}{\rho} \frac{\partial p}{\partial x} \right) \left( s \frac{\sigma_z}{\sigma_u} \right) + \frac{\partial^2 w}{\partial y^2} + \frac{B_0}{\rho} \left[ \sigma_y (E_x - w B_y) + \sigma_z (E_z + u B_y) \right] = V \frac{\partial w}{\partial y}
\]
(3)

The slip boundary conditions are

\[
u = \beta \frac{du}{dy} \quad \text{and} \quad w = \beta \frac{dw}{dy} \quad \text{at} \quad y = -h
\]
(4)
\[ u = -\beta \frac{du}{dy} \quad \text{and} \quad w = -\beta \frac{dw}{dy} \quad \text{at} \quad y = h \]

Separating equns. (3.2) to (3.5) into a steady part denoted by a bar (\(\bar{\cdot}\)) and an unsteady part denoted by a tilde (\(\tilde{\cdot}\)), by using \(u(y, t) = \bar{u}(y) + \tilde{u}(y, t)\) and \(w(y, t) = \bar{w}(y) + \tilde{w}(y, t)\), we get

**Steady part**

\[
\left( \frac{1}{\rho} \frac{\partial p}{\partial x} \right) \left( 1 - s \left( 1 - \frac{\sigma_z}{\sigma_o} \right) \right) + \nu \frac{\partial^2 u}{\partial y^2} + \frac{B_o}{\rho} \left[ -\sigma_1(\overline{E_z} + \bar{u}B_o) + \sigma_2(\overline{E_z} - \bar{w}B_o) \right] = V \frac{\partial \bar{u}}{\partial y} - g \sin \theta
\]

\[
\left( \frac{1}{\rho} \frac{\partial p}{\partial x} \right) s \frac{\sigma_z}{\sigma_o} + \nu \frac{\partial^2 w}{\partial y^2} + \frac{B_o}{\rho} \left[ \sigma_1(\overline{E_z} - \bar{w}B_o) + \sigma_2(\overline{E_z} + \bar{u}B_o) \right] = V \frac{\partial \bar{w}}{\partial y}
\]

\[
\bar{u} = \beta \frac{\partial \bar{u}}{\partial y} \quad \text{and} \quad \bar{w} = \beta \frac{\partial \bar{w}}{\partial y} \quad \text{at} \quad y = -h
\]

\[
\tilde{u} = -\beta \frac{\partial \tilde{u}}{\partial y} \quad \text{and} \quad \tilde{w} = -\beta \frac{\partial \tilde{w}}{\partial y} \quad \text{at} \quad y = h
\]

**Unsteady part**

\[
\left( \frac{1}{\rho} \frac{\partial p}{\partial x} \right) \left( 1 - s \left( 1 - \frac{\sigma_z}{\sigma_o} \right) \right) + \nu \frac{\partial^2 u}{\partial y^2} + \frac{B_o}{\rho} \left[ -\sigma_1(\overline{E_z} + \bar{u}B_o) + \sigma_2(\overline{E_z} - \bar{w}B_o) \right] = V \frac{\partial \bar{u}}{\partial y} + \frac{\partial \tilde{u}}{\partial t}
\]

\[
\left( \frac{1}{\rho} \frac{\partial p}{\partial x} \right) s \frac{\sigma_z}{\sigma_o} + \nu \frac{\partial^2 w}{\partial y^2} + \frac{B_o}{\rho} \left[ \sigma_1(\overline{E_z} - \bar{w}B_o) + \sigma_2(\overline{E_z} + \bar{u}B_o) \right] = V \frac{\partial \bar{w}}{\partial y} + \frac{\partial \tilde{w}}{\partial t}
\]

\[
\tilde{u} = \beta \frac{\partial \tilde{u}}{\partial y} \quad \text{and} \quad \tilde{w} = \beta \frac{\partial \tilde{w}}{\partial y} \quad \text{at} \quad y = -h
\]

\[
\bar{u} = -\beta \frac{\partial \bar{u}}{\partial y} \quad \text{and} \quad \bar{w} = -\beta \frac{\partial \bar{w}}{\partial y} \quad \text{at} \quad y = h
\]

### 2. Non-dimensionalization of the flow quantities

The following non-dimensional quantities are introduced to make the basic equations and the boundary conditions dimensionless.

**Steady part**

\[
\bar{u} = \frac{\bar{u}}{\left( A_h \right)}, \quad \bar{w} = \frac{\bar{w}}{\left( A_h \right)}, \quad \bar{m}_x = \frac{\bar{E}_x}{\left( B_o A_h \right)} \quad \bar{m}_z = \frac{\bar{E}_z}{\left( B_o A_h \right)}
\]

\[
\bar{u} = \frac{\bar{u}}{h}, \quad \bar{w} = \frac{\bar{w}}{h}, \quad \beta = \frac{\beta}{h}, \quad A = \frac{1}{\rho} \frac{\partial p}{\partial x}, \quad A_1 = -A, \quad R = \frac{V h}{\nu A_1} = F_1
\]

**Unsteady part**

\[
\bar{u} = \frac{\bar{u}}{\left( A_h \right)}, \quad \bar{w} = \frac{\bar{w}}{\left( A_h \right)}, \quad \bar{m}_x = \frac{\bar{E}_x}{\left( B_o A_h \right)} \quad \bar{m}_z = \frac{\bar{E}_z}{\left( B_o A_h \right)}
\]

\[
\bar{u} = \frac{\bar{u}}{h}, \quad \bar{w} = \frac{\bar{w}}{h}, \quad \beta = \frac{\beta}{h}, \quad B_1 = -B, \quad \frac{\omega}{\nu A_1} = F_1
\]

\[
M = \frac{B_o h}{\rho \nu} \quad \sigma = \frac{E_s}{B_o u_p}, \quad m = \frac{E_z}{B_o u_p} \quad \sigma = \frac{1}{1 + m}, \quad \sigma = \frac{1}{1 + m^2}
\]
\[ L_1 = 1 - s \left[ 1 - \frac{1}{1 + m^2} \right], \quad L_2 = \frac{-sm}{1 + m^2} \]

In view of the above dimensionless quantities, eqns. (6) to (13) reduce to the following form: Neglecting the asterisks (*), we get the fundamental equations and boundary conditions as

**Steady part**

\[
L_1 + \frac{1}{R} \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{M^2}{R} \left[ - \frac{\sigma_1}{\sigma_0} \left( \bar{m}_t + \bar{u} \right) + \frac{\sigma_2}{\sigma_0} \left( \bar{m}_z - \bar{w} \right) \right] = \frac{\partial \bar{u}}{\partial y} - F_1 \sin \theta
\]

(14)

\[
L_2 + \frac{1}{R} \frac{\partial^2 \bar{w}}{\partial y^2} + \frac{M^2}{R} \left[ - \frac{\sigma_1}{\sigma_0} \left( \bar{m}_z - \bar{w} \right) + \frac{\sigma_2}{\sigma_0} \left( \bar{m}_z + \bar{u} \right) \right] = \frac{\partial \bar{w}}{\partial y}
\]

(15)

\[
\bar{u} = \beta \frac{d\bar{u}}{dy} \quad \text{and} \quad \bar{w} = \beta \frac{d\bar{w}}{dy} \quad \text{at} \quad y = -1
\]

(16)

\[
\bar{u} = -\beta \frac{d\bar{u}}{dy} \quad \text{and} \quad \bar{w} = -\beta \frac{d\bar{w}}{dy} \quad \text{at} \quad y = 1
\]

(17)

**Unsteady part**

\[
e^{i\omega t} L_1 + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{M^2}{R} \left[ - \frac{\sigma_1}{\sigma_0} \left( \bar{m}_t + \bar{u} \right) + \frac{\sigma_2}{\sigma_0} \left( \bar{m}_z - \bar{w} \right) \right] = R \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{u}}{\partial t}
\]

(18)

\[
e^{i\omega t} L_2 + \frac{\partial^2 \bar{w}}{\partial y^2} + \frac{M^2}{R} \left[ - \frac{\sigma_1}{\sigma_0} \left( \bar{m}_z - \bar{w} \right) + \frac{\sigma_2}{\sigma_0} \left( \bar{m}_z + \bar{u} \right) \right] = R \frac{\partial \bar{w}}{\partial y} + \frac{\partial \bar{w}}{\partial t}
\]

(19)

\[
\bar{u} = \beta \frac{d\bar{u}}{dy} \quad \text{and} \quad \bar{w} = \beta \frac{d\bar{w}}{dy} \quad \text{at} \quad y = -1
\]

(20)

\[
\bar{u} = -\beta \frac{d\bar{u}}{dy} \quad \text{and} \quad \bar{w} = -\beta \frac{d\bar{w}}{dy} \quad \text{at} \quad y = 1
\]

(21)

For simplicity, we introduce the complex notations as

\[
\bar{q} = \bar{u} + i\bar{w}, \quad \tilde{q} = \bar{u} + i\tilde{w}, \quad L = L_1 + iL_2, \quad \bar{E} = \bar{m}_t + i\bar{m}_z, \quad \tilde{E} = \bar{m}_z + i\tilde{m}_t
\]

Equations (13) to (21) can be written in complex form as

**Steady part**

\[
\frac{\partial^2 \bar{q}}{\partial y^2} - R \frac{\partial \bar{q}}{\partial y} + a \bar{q} = -LR - RF_1 \sin \theta + iaRE
\]

(22)

\[
\bar{q} = \beta \frac{d\bar{q}}{dy} \quad \text{at} \quad y = -1
\]

(23)

\[
\bar{q} = -\beta \frac{d\bar{q}}{dy} \quad \text{at} \quad y = 1
\]

(24)

**Unsteady part**

\[
\frac{\partial^2 \tilde{q}}{\partial y^2} - R \frac{\partial \tilde{q}}{\partial y} + a \tilde{q} = ia\tilde{E} - e^{i\omega t} L + \frac{\partial \tilde{q}}{\partial t}
\]

(25)

\[
\tilde{q} = \beta \frac{d\tilde{q}}{dy} \quad \text{at} \quad y = -1
\]

(26)
\[ \tilde{q} = -\beta \frac{d\tilde{q}}{dy} \quad \text{at} \quad y = 1 \]  

(27)

where \( a = a_1 + ia_2 \), \( a_1 = \frac{M^2}{1 + m^2} \), \( a_2 = \frac{M^2 m}{1 + m^2} \).

3. Solution of the problem

If the side walls are made up of conducting material and short circuited by an external conductor, the induced electric current flows out of the channel. In this case no electric potential ts are obtained as follows:

Steady part

\[ \frac{\partial^2 \tilde{q}}{\partial y^2} - R \frac{\partial \tilde{q}}{\partial y} + a\tilde{q} = k \]  

(28)

Solving equation (28) under the boundary conditions (23) and (24), we get the velocity field as

\[ \tilde{q} = C_1 \cosh m_1 y + \frac{k}{a} \]  

(29)

where

\[ C_1 = -\frac{k}{ab_1} \]

\[ a = a_1 + ia_2, \quad a_1 = \left[ \left( -\frac{\sigma_1}{\sigma_0} M^2 \right) \right], \quad a_2 = \left[ \left( \frac{\sigma_2}{\sigma_0} M^2 \right) \right] \]

\[ k = k_1 + ik_2, \quad k_1 = -R (L_1 + F_1 \sin \theta), \quad k_2 = -RL_2, \quad L = L_1 + iL_2 \]

\[ b_1 = \cosh m_1 + m_1 \beta \sinh m_1, \quad b_2 = -\sinh m_2 - m_2 \beta \cosh m_2, \quad m_{1,2} = \frac{R \pm \sqrt{R^2 - 4a}}{2} \]

Separating equation (29) into real and imaginary parts, the primary velocity \( \tilde{u} \) and secondary velocity \( \tilde{w} \) in steady part are given by

\[ \tilde{u} = \text{Re} \tilde{q} \]

\[ \tilde{w} = \text{Im} \tilde{q} \]

Unsteady part

\[ \frac{\partial^2 \tilde{q}}{\partial y^2} - R \frac{\partial \tilde{q}}{\partial y} + a\tilde{q} = \frac{\partial \tilde{q}}{\partial t} - e^{i\omega t} L \]  

(30)

Letting \( \tilde{q} = f(y)e^{i\omega t} \) in equation (30), we get

\[ f''(y) - Rf'(y) + df(y) = -L \]  

(31)

Solving equation (31) under the boundary conditions (26) and (27), we get

\[ f(y) = C_3 \cosh m_3 y - \frac{L}{d} \]  

(32)

The unsteady part of the velocity is given by

\[ \tilde{q} = f(y)e^{i\omega t} \]  

(33)

Separating equation (33) into real and imaginary parts, the primary velocity \( \tilde{u} \) and secondary velocity \( \tilde{w} \) in unsteady part are given by
\[ \begin{align*}
\vec{u} &= \text{Re} \quad \vec{q} \\
\vec{w} &= \text{Im} \quad \vec{q}
\end{align*} \]

3.1 Mass Flux

The instantaneous mass flux \( F \) is given by

\[
F = \left[ \int_{-1}^{1} f(y) dy \right] e^{i \omega t} 
= e^{i \omega t} \left[ \frac{L}{db, m} - 2 \sinh m, \frac{2L}{d} \right] 
= \frac{2L e^{i \omega t}}{d} \left[ \sinh m, \frac{b, m}{b, m} - 1 \right]
\]

where

\[
C_3 = \frac{L}{db, m}, \quad d = a - i \omega
\]

\[
b_1 = \cosh m, + \beta m, \sinh m, \quad b_4 = -\sinh m, - \beta m, \cosh m, m, m_3, m_4 = \frac{R \pm \sqrt{R^2 - 4d}}{2}
\]

4. Results and discussion

Velocity profiles for the pulsatile flow of an ionized gas between two porous plates in both cases (partially ionized \( s=0 \) and fully ionized \( s=0.5 \)) for steady and unsteady parts are depicted graphically in figures 2 to 12 for various values of \( M, m, R, \beta, \theta, F, \) and \( \omega t \).

The variation in primary and secondary velocities for steady part are shown in figures 2, 3 and unsteady part are shown in figures 8, 9 respectively for different values of Hartmann number \( M \) and for fixed \( m, R, \beta, \theta, F, \) and \( \omega t \). It is noticed that both primary and secondary velocities decrease with the increase in the Hartmann number \( M \). This is because the increase in the magnetic field gives rise to reduction in velocity in the permeable beds.

The variation in primary and secondary velocities for the steady part are shown in figures 4, 5 and unsteady part are shown in figures 10, 11 respectively for different values of Hall parameter \( m \) and for fixed \( M, R, \beta, \theta, F, \) and \( \omega t \). It is noticed that both primary and secondary velocities increase with the increase in the Hall parameter \( m \).

The variation in primary and secondary velocities for steady part are shown in figures 6, 7 and unsteady part are shown in figures 12, 13 respectively for different values of slip parameter \( \beta \) and for fixed \( M, m, R, \theta, F, \) and \( \omega t \). It is noticed that for steady part, primary velocity decreases at the centre of the channel but increases at both the walls and secondary velocity increases with the increase in the slip parameter \( \beta \). It is also noticed that for unsteady part, both primary and secondary velocities increase with an increase in the slip parameter \( \beta \).

Graphs for steady part

![Fig 2: Primary velocity distribution for different values of M with s = 0, s = 0.5, m = 1.5, R = 1, \beta = 0.1, \theta = \pi/6, F_1 = 0.5](image)

![Fig 3: Secondary velocity distribution for different values of M with s = 0, s = 0.5, m = 1.5, R = 1, \beta = 0.1, \theta = \pi/6, F_1 = 0.5](image)
Fig 4: Primary velocity distribution for different values of $m$ with $s = 0, s = 0.5, M = 5, R = 1, \beta = 0.1$

\[ \theta = \frac{\pi}{6}, F_1 = 0.5 \]

Fig 5: Secondary velocity distribution for different values of $m$ with $s = 0, s = 0.5, M = 5, R = 1, \beta = 0.1$

\[ \theta = \frac{\pi}{6}, F_1 = 0.5 \]

Fig 6: Primary velocity distribution for different values of $\beta$ with $s = 0, s = 0.5, M = 5, m = 1.5, R = 1,$

\[ \theta = \frac{\pi}{6}, F_1 = 0.5 \]

Fig 7: Secondary velocity distribution for different values of $\beta$ with $s = 0, s = 0.5, M = 5, m = 1.5, R = 1,$

\[ \theta = \frac{\pi}{6}, F_1 = 0.5 \]

Graphs for unsteady part

Fig 8: Primary velocity distribution for different values of $M$ with $s = 0, s = 0.5, m = 1.5, R = 1, \beta = 0.1$

\[ \omega = \frac{\pi}{3}, t = 5 \]

Fig 9: Secondary velocity distribution for different values of $M$ with $s = 0, s = 0.5, m = 1.5, R = 1, \beta = 0.1$

\[ \omega = \frac{\pi}{3}, t = 5 \]
Fig 10: Primary velocity distribution for different values of $m$ with $s = 0, s = 0.5, M = 5, R = 1, \beta = 0.1$

$\omega = \frac{\pi}{3}, t = 5$

Fig 11: Secondary velocity distribution for different values of $m$ with $s = 0, s = 0.5, M = 5, R = 1, \beta = 0.1$

$\omega = \frac{\pi}{3}, t = 5$

Fig 12: Primary velocity distribution for different values of $\beta$ with $s = 0, s = 0.5, M = 5, m = 1.5, R = 1$

$\omega = \frac{\pi}{3}, t = 5$

Fig 13: Secondary velocity distribution for different values of $\beta$ with $s = 0, s = 0.5, M = 5, m = 1.5, R = 1$

$\omega = \frac{\pi}{3}, t = 5$

5. References