

A Non-Newtonian mathematical model for two phase hepatic blood flow by using Herschel Bulkley law in Venules with special reference to hepatitis B

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In this paper, we have presented a model of two phased hepatic blood flow in Hepatic venules remote from the heart and proximate to the Liver keeping in view the nature of Hepatic blood circulation in human body. As the hematocrit increases, the blood in the venules remote from the heart shows Herschel Bulkley Law of non-Newtonian flow. P.N. Pandey and V. Upadhyay have considered that the blood flow has two phased, one of which that of red blood cells and other is plasma. They have also applied the power law model in Bio-fluid mechanical setup. We have collected a clinical data in case of Hepatitis B for hemoglobin versus blood pressure. The overall presentation is in tensorial form and the solution technique adopted is analytical as well as numerical. The role of hematocrit is explicit in the determination of blood pressure drop in case of Hepatic disease Hepatitis B.

Keyword: Structure & function of the Liver and hepatic venules, Hepatic Blood Flow, Non-Newtonian Herschel Bulkley Law

INTRODUCTION

Structure and function of Liver

The liver is one of the largest, anatomically and functionally most complex organ of the human body. The liver constitutes 2.5% of the human body weight and is the largest organ in the body [1, 2]. The liver is a reddish-brown wedge-shaped organ with four lobes of unequal size and shape. Four lobes of man liver are right lobe, left lobe quadrats and caudate lobe. Liver lobes are sarrounded by a thick capsule, mostly overlaid with reflected periforinum [3].

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A human liver normally weighs 1.44–1.66 kg (3.2–3.7 lb) [4] and has a width of about 15 cm. Located in the right upper quadrant of the abdominal cavity, it rests just below the diaphragm, to the right of the stomach and overlies the gallbladder [5].

The liver is connected to two large blood vessels: the hepatic artery and the portal vein. The hepatic artery carries oxygen-rich blood from the aorta, whereas the portal vein carries blood rich in digested nutrients from the entire gastrointestinal tract and also from the spleen and pancreas. These blood vessels subdivide into small capillaries known as liver sinusoids, which then lead to a lobule. Lobules are the functional units of the liver.

Each lobule is made up of millions of hepatic cells (hepatocytes) which are the basic metabolic cells. The lobules are held together by a fine dense irregular fibroelastic connective tissue layer which extends into the structure of the liver, by accompanying the vessels (veins and arteries), ducts and nerves through the hepatic portal, as a fibrous capsule called Glisson's capsule.

The liver has a wide range of functions, including detoxification of various metabolites and toxic matter, regulation of glycogen storage, decomposition of red blood cells, which are responsible for carrying oxygen around the body, hormone production and the production of biochemical's necessary for digestion. The liver breaks down ammonia into urea as part of the urea cycle, and the urea is excreted in the urine. The liver is the only human internal organ capable of natural regeneration of lost tissue; as little as 25% of a liver can regenerate into a whole liver [6]. Although there are many studies of hepatic function and ageing, results have been conflicting or unsubstantiated. Liver weight and volume decrease with age, and liver blood flow is reduced [7].

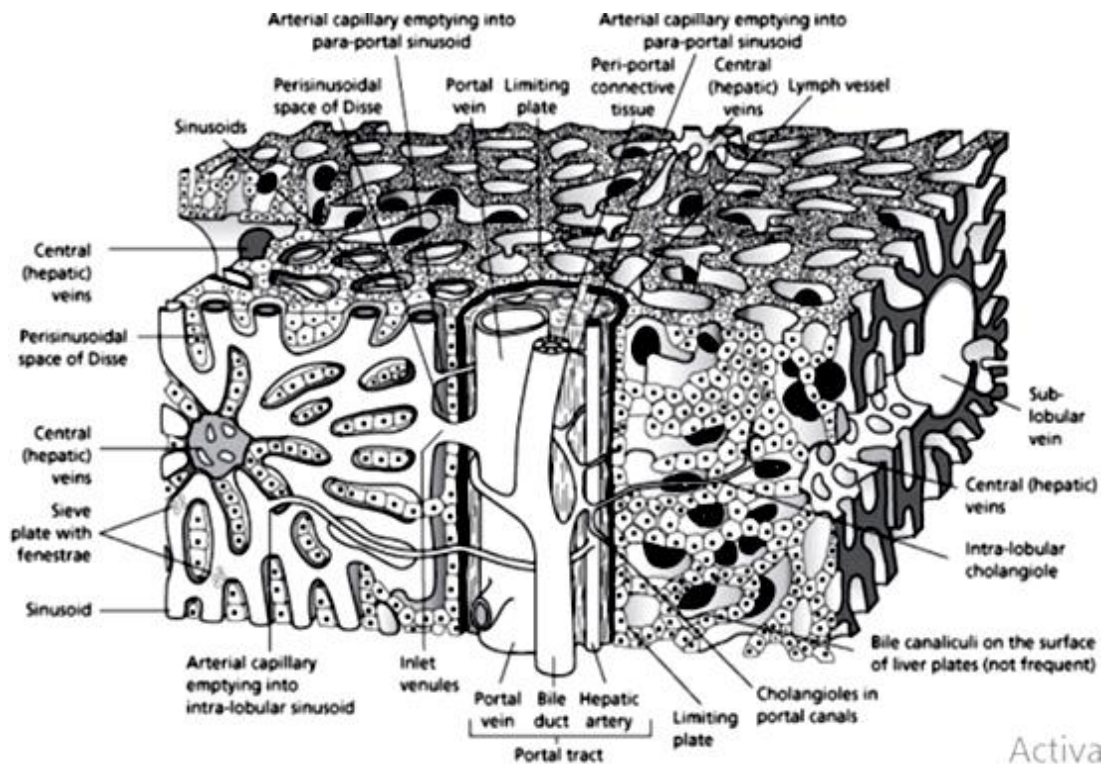


Fig 1.1: Normal Liver Hipatology [8]

Structure and function of Hepatic Venules

A venule is a very small blood vessel in the microcirculation that allows blood to return from the capillary beds to drain into the larger blood vessels, the veins. Venules range from 7 to 50 μm in diameter. Veins contain approximately 70% of total blood volume, 25% of which is contained in the venules [9]. Many venules unite to form a vein.

Blood flows through the liver tissue and empties into the central vein of each lobule. The central veins coalesce into hepatic veins that collect the blood leaving the liver and bring it to the heart.

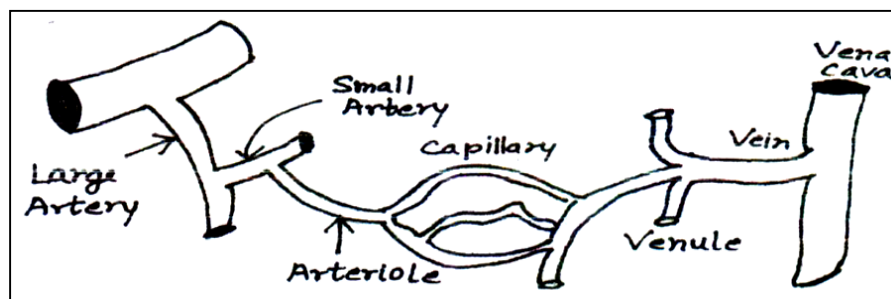


Fig 1.2: Major types of blood vessels

The hepatic veins are the veins that drain de-oxygenated blood from the liver into the inferior vena cava. There are usually three upper hepatic veins draining from the left, middle, and right parts of the liver. These are larger than the group of lower hepatic veins that can number from six to twenty. All the veins empty into the inferior vena cava at the back of the liver^[10]. The large hepatic veins arise from smaller veins found within the liver, and ultimately from numerous central veins of the liver lobules. None of the hepatic veins have valves.

The portal vein or hepatic portal vein is a blood vessel that carries blood from the gastrointestinal tract, gallbladder, pancreas and spleen to the liver. This blood is rich in nutrients that have been extracted from food, and the liver processes these nutrients; it also filters toxins that may have been ingested with the food. 75% of total liver blood flow is through the portal vein, with the remainder coming from the hepatic artery proper. The blood leaves the liver to the heart in the hepatic veins. Measuring approximately 8 cm (3 inches) in adults^[11].

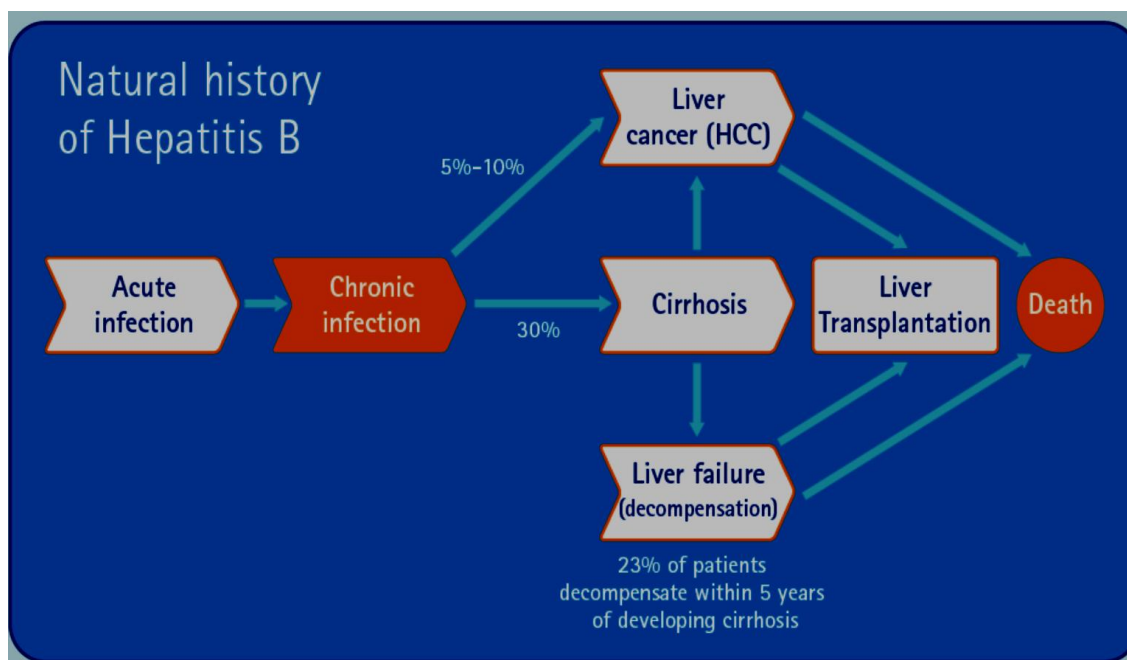
In the hepatic portal system, the liver receives a dual blood supply from the hepatic portal vein and hepatic arteries. The hepatic portal vein carries venous blood drained from the spleen, gastrointestinal tract and its associated organs; it supplies approximately 75% of the liver's blood. The hepatic arteries supply arterial blood to the liver and account for the remainder of its blood flow.

The velocity of blood flow decreases successively because of the fact that hepatic veinules are relatively a far away from the heart. Hence the pumping of the heart on these vessels is relatively low^[12]. Secondly these vessels are relatively narrow down more rapidly. In this situation, the blood cells line up on the axis to build up rouleaux. Hence a yield stress is produced and we have found a linear relation between hematocrit and blood pressure drop in hepatic veinules.

Hepatitis B and Its Virus

In 1947 Mac Callum classified viral Hepatitis A & viral hepatitis B. Hepatitis B virus was discovered in 1965 by Dr. Baruch Blumberg.

In 1981 The FDA approved sophisticated Plasma, Derived Hepatitis vaccine are human use. In 2001, FDA approved a combination Hepatitis A and Hepatitis B vaccine (Twinrix, Glaxo Smithkline). During 1990-2004, incidence of acute Hepatitis B in the United States declined 75%. The greatest decline (94%) occurred among children and adolescent. People with chronic hepatitis B infection are at increased risk of dying from cirrhosis and liver cancer. Many Cancer Patients at Risk for Hepatitis B Virus Reactivation^[13]. According to an estimation of the World Health Organization (WHO), which says that two to five percent of India's population may be affected by the virus. Each year, one lakh patients die of viral Hepatitis (including Hepatitis B and C) in India.



Adapted from Torresi *et al*, 2000 and Fattovich *et al*, 2003.

Real Model

If we want to apply mathematics in bio-physical problem, then real model is considered as:

Choice of frame of reference

We have to select a frame of reference for mathematical modeling of the state of a moving blood-keeping in view the difficulty and generality of the problem of blood flow, we select generalized three-dimensional orthogonal curvilinear co-ordinate system, briefly prescribed as E3, called as 3-dim Euclidean space, We interpret the quantities related to blood flow in tensorial form which is comparatively more realistic.

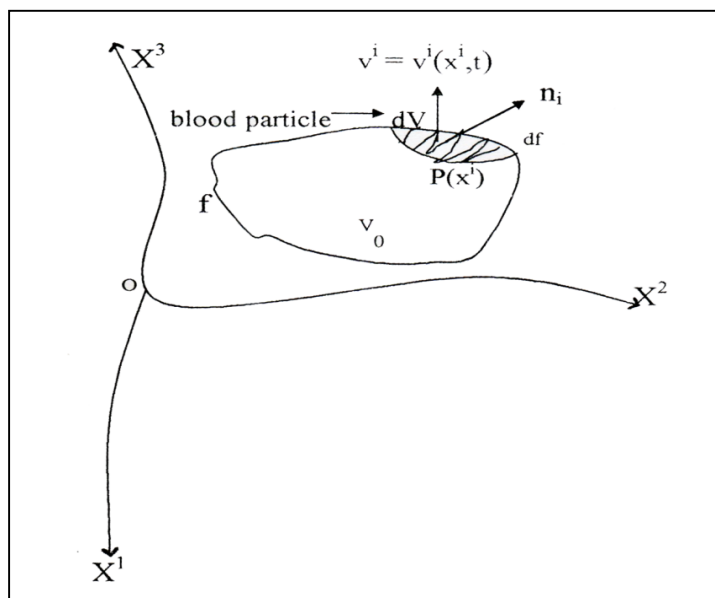


Fig 2.1: Frame of Reference

Now, let the co-ordinate axes be OX^i where O is origin and superscript $i = 1, 2, 3$. let X^i be the co-ordinates of any point P in space.

The biophysical laws thus expressed fully hold good in any co-ordinate system, which is compulsion for the truthfulness of the laws (1990) [14].

Choice of parameters

The mathematical description of the state if a moving blood is affected by means of functions which give the distribution of the blood velocity $v^k = v^k(X^i, t)$, $k = 1, 2, 3$ and of any two thermodynamic quantities pertaining to the blood, for instance the pressure $P = P(X^i, t)$ and the density $\rho = \rho(X^i, t)$. As is well known, all the thermodynamic quantities are determined by the values of any two of them and temperature $T = T(X^i, t)$. But our observation is at constant temperature, together with the equation of state. Hence, if we are given five quantities, namely the three components of velocity v^k , the pressure P and the density ρ , the state of moving blood is completely determined.

All these quantities are, in general, functions of the co-ordinates X^i , $i = 1, 2, 3$ and of the time t. We emphasize that $v^k(X^i, t)$ is the velocity of the blood at a given point X^i in space and at a given t, i.e. it refers to fixed points in space and not to fixed particles of the blood; in the course of time, the latter move about in space. The same remarks apply to P and ρ . Blood is a mixed fluid,

Hypothesis of two phase blood flow

Mainly there are two-phases in blood, The first phase is plasma, while the other phase is that of blood cells. The blood cells are enclosed with a semi-permeable membrane whose density is greater than that of plasma, these blood cells are uniformly distributed in plasma, Thus blood can be considered as a homogeneous mixture of two-phases [15].

Constitutive forms of non – Newtonian blood flow

It is well known that, blood is non-homogeneous mixture of plasma and blood cells. Though for practical purposes it may be considered to be homogeneous two- phased non-Newtonian mixture of plasma and blood cells, at low shear rates and during its flow through narrow blood vessels. The blood plasma may be separately considered to be Newtonian with a viscosity of about 1.5 times that of water. Hence for low hematocrit, the homogeneous mixture of blood may be treated as Newtonian. In general, stress tensor τ and strain - rate tensor e both have six different components. If the functional relations between the components of the two tensors is linear, the blood is called Newtonian, otherwise non-Newtonian. In the situation when blood behave like non-Newtonian, we can measure effective viscosity, which is depend on strain rate tensor. The constitutive equations for whole blood-mixture are as follows:

(i) Newtonian equation

$$\tau = \eta e \text{ where } \eta \text{ is the viscosity coefficient.}$$

This is found to hold good in the broad blood vessels, where there is low hematocrit [16].

(ii) The non-Newtonian power law equation

$$\tau = \eta e^n$$

This is found to be conformable for strain rate between 5 and 200 sec^{-1} , $0.68 \leq n \leq 0.80$ [17].

(iii) The non-Newtonian Herschel-Bulkley equation [18].

$$\tau = \eta e^n + \tau_0 \quad (\tau \geq \tau_0) \quad e = 0 \quad (\tau < \tau_0)$$

It holds good when blood shows yield stress τ_o . We notice that the yield stress arises because blood cells form aggregates in the form of rouleaux at low strain rate.

If $\tau < \tau_o$, no blood flow-takes place. It is found that yield stress is given by the following formula :

$\tau_o^{1/3} = \frac{A(H-H_m)}{100}$, where $A = (0.008 \pm 0.002 \frac{\text{dyne}}{\text{cm}^2})^{1/3}$, H is normal hematocrit and H_m is the hematocrit below which there is no yield stress.

Mathematical Modeling

As the velocity of Blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively because of the fact that arterioles, veinules and veins these vessels are relatively a far enough from the heart. Hence the pumping of the heart on these vessels is relatively low [19]. Secondly these vessels relatively narrow down more rapidly. In this situation, the blood cells line up on the axis to build up rouleaux. Hence a yield stress is produced.

The Herschel Bulkley law holds good on the two phase blood flow through veins arterioles, veinules and whose constitutive equation is as follows:

$T' = \eta_m e^n + T_p$ ($T' \geq T_p$) and $e = 0$ ($T' < T_p$) where, T_p is the yield stress.

When strain rate $e = 0$ ($T' < T_p$) a core region is formed which flows just like a plug. Let the radius of the plug be r_p . The stress acting on the surface of plug will be T_p . Equating the forces acting on the plug, we get [fig (3)]

$$\pi r_p^2 T_p = T_p 2\pi r_p \Rightarrow r_p = 2 \frac{T_p}{P} \tag{3.1}$$

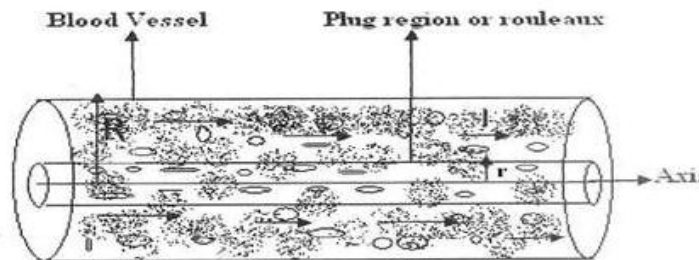


Fig (3): Herschel Bulkley blood flow

The Constitutive equation for test part of the blood vessel is

$$T' = \eta_m e^n + T_p \text{ or } T' - T_p = \eta_m e^n = T_e$$

Where, T_e = effective Stress, whose generalized form will be as follows”

$$T^{ij} = -P g^{ij} + T_e^{ij} \text{ where, } T_e^{ij} = \eta_m (e^{ij})^n \text{ while } e^{ij} = g^{jk} V_k^i$$

Where, the symbols have their usual meanings.

Now we describe the basic equations for Herschel Bulkley blood flow as follows:

Equation of Continuity

$$\frac{1}{\sqrt{g}} (\sqrt{g} V^i)_{,i} = 0$$

Equation of Motion

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^i v^i_{;j} = - T_{e,j}{}^{;i} \tag{3.2}$$

Where all the symbols have their usual meanings.

Analysis (Solution)

The basic equations for Herschel Bulkley blood flow as follows:

The constitutive equation for rest part of blood vessel is

$$T' = \eta_m e^n + T'_0$$

Whose generalized form will be as follows:

$$T^{ij} = -p g^{ij} + T'{}^{ij} \tag{4.1}$$

Equation of Continuity-

$$\frac{1}{\sqrt{g}} (\sqrt{g} v^i)_{;i} = 0 \tag{4.2}$$

Equation of Motion -

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v^i_{;j} = - T_{,j}{}^{;i} \tag{4.3}$$

Where all the symbols have their usual meanings.

The above equations are transformed into cylindrical form so as to solve them as power law model (4.1) to get

$$- \frac{dv}{dr} = \left(\frac{\frac{1}{2} P r - T_p}{\eta_m} \right)^{\frac{1}{n}} \tag{4.4}$$

Substituting the value of T_p into (4.4), we get

$$\begin{aligned} - \frac{dv}{dr} &= \left(\frac{\frac{1}{2} P r - \frac{1}{2} P r_p}{\eta_m} \right)^{\frac{1}{n}} \\ \Rightarrow \frac{dv}{dr} &= - \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (r - r_p)^{\frac{1}{n}} \end{aligned} \tag{4.5}$$

Integrating above equation (4.5) under the no slip boundary condition: $v = 0$ at $r = R$, So we get:

$$v = \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} \frac{n}{n+1} \left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1} \right] \tag{4.6}$$

Which is the formula for velocity of blood flow in arterioles, veinules and veins.

Putting $r = r_p$ to get the velocity v_p of plug flow as follows:

$$v_p = \frac{n}{n+1} \left(\frac{P}{2\eta_m} \right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} \tag{4.7}$$

Where the value of r_p is $\frac{2T_p}{P}$.

Result & Discussion

(Bio-Physical Interpretation)

The flow flux of two phased blood flow in arterioles, veinules and veins is

$$\begin{aligned}
 Q &= \int_0^{r_p} 2\pi r v_p dr + \int_{r_p}^R 2\pi r v dr \\
 &= \int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} dr \\
 &+ \int_{r_p}^R 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} [(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}] dr \tag{5.1}
 \end{aligned}$$

Using v_p and v , we get

$$\begin{aligned}
 Q &= \frac{2\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} \left[\frac{r^2}{2} \right]_0^{r_p} + \frac{2\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \left[\frac{r^2}{2} (R - r_p)^{\frac{1}{n}+1} - \frac{r(r-r_p)^{\frac{1}{n}}}{\frac{1}{n}+2} + \frac{(r-r_p)^{\frac{1}{n}+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} \right]_{r_p}^R \\
 &= \frac{2\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \left[r_p^2 (R - r_p)^{\frac{1}{n}+1} + R^2 (R - r_p)^{\frac{1}{n}+1} - \frac{2R(R-r_p)^{\frac{1}{n}+2}}{(\frac{1}{n}+2)} + \frac{2(R-r_p)^{\frac{1}{n}+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} \right. \\
 &\quad \left. - r_p^2 (R - r_p)^{\frac{1}{n}+1} \right] \\
 &= \frac{\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} R^{\frac{1}{n}+3} \left[\frac{r_p^2}{R^2} \left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+1} + \left(1 + \frac{r_p}{R}\right) \left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2} \right. \\
 &\quad \left. - \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2}}{(\frac{1}{n}+2)} + \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} \right]
 \end{aligned}$$

Now let $R = 1$ and $r_p = \frac{1}{3}$ i.e. $\frac{r_p}{R} = \frac{1}{3}$, we have

$$\begin{aligned}
 Q &= \frac{\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (1) \left[\frac{1}{9} \left(1 - \frac{1}{3}\right)^{\frac{1}{n}+1} + \left(1 + \frac{1}{3}\right) \left(1 - \frac{1}{3}\right)^{\frac{1}{n}+2} - \frac{2\left(1 - \frac{1}{3}\right)^{\frac{1}{n}+2}}{\frac{1}{n}+2} \right. \\
 &\quad \left. + \frac{2\left(1 - \frac{1}{3}\right)^{\frac{1}{n}+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} \right] \\
 &= \frac{\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \left[\frac{1}{9} \left(\frac{2}{3}\right)^{\frac{1}{n}} \cdot \frac{2}{3} + \left(\frac{4}{3}\right) \left(\frac{2}{3}\right)^{\frac{1}{n}} \frac{4}{9} - \frac{2\left(\frac{2}{3}\right)^{\frac{1}{n}} \cdot 4}{\frac{1}{n}+2} + \frac{2\left(\frac{2}{3}\right)^{\frac{1}{n}}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)} \cdot \frac{8}{27} \right] \\
 Q &= \frac{\pi n}{(n+1)} \left(\frac{P}{2\eta_m} \cdot \frac{2}{3}\right)^{\frac{1}{n}} \frac{2}{27} \left[\frac{26n^2+33n+9}{6n^2+5n+1} \right]
 \end{aligned}$$

$$\frac{27 \times Q}{2\pi} = \left(\frac{P}{3\eta_m}\right)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right] \quad (5.2)$$

Where Q = Flow flux η_m = Viscosity of Mixture
 n = Parameter P = Pressure Gradient

Observations: Hemoglobin Vs. Blood pressure is taken from Kanpur Medical Centre, Kanpur by Dr Gaurav Chawla.

Patient's Name: Sanjay Gupta Age/Sex: 36/Male
 Ref By: Dr Gaurav Chawla I.P. No. : 180/2012

S. No	Date	B.P. (In mm hg)	Hemoglobin (gm / dl)	Hematocrit H	B.P. (In Pascal)
1	05.02.12	140/90	7.9	23.7	18664.8/11998.8
2	06.02.12	150/90	8.1	24.3	19998.0/11998.8
3	07.02.12	164/97	8.5	25.5	21864.5/12932.0
4	08.02.12	143/93	8.0	24.0	19064.8/12398.8
5	09.02.12	130/90	7.5	22.5	17331.6/11998.8
6	10.02.12	140/80	7.0	21.0	18664.8/10665.6

Average Hematocrit = 23.5
 Average systolic blood pressure = 19264.75 pa
 Average diastolic blood pressure = 11998.8 pa

$$\text{Pressure on veinules} = \frac{2}{3} \left[\frac{\left(\frac{S+D}{2}\right) + D}{3} \right]$$

$$P_i = \frac{2}{3} \left[\frac{\left(\frac{19264.75 + 11998.8}{2}\right) + 11998.8}{3} \right] = 6140.13 \text{ pa}$$

Pressure on veins = $\frac{1}{3}$ (Pressure on veinules)

$$P_f = \frac{1}{3} (6140.13) = 2046.71 \text{ pa}$$

Length of hepatic portal vein ($Z_f - Z_i$) = 0.08 m^[20]

Viscosity of mixture (blood) = 0.0035 p.s.^[21]

Viscosity of plasma = 0.0015 p.s.^[22]

Flow flux of blood = 0.01666 litre/sec

Now we know that

$$\eta_m = \eta_c X + \eta_p (1 - X)$$

$$\text{or } \eta_m = \eta_c \frac{H}{100} + \eta_p \left(1 - \frac{H}{100}\right) \text{ Where } X = \frac{H}{100}$$

$$0.0035 = \eta_c \left(\frac{23.5}{100}\right) + (0.0015) \left(1 - \frac{23.5}{100}\right)$$

$$\eta_c = \frac{0.0023525}{0.235} = 0.01001$$

or η_c = Viscosity of Cells = 0.01001 p.s.

Now putting the value of η_c in η_m , We have

$$\eta_m = 0.01001 \left(\frac{H}{100}\right) + 0.0015 \left(1 - \frac{H}{100}\right)$$

$$\eta_m = (8.51 \times 10^{-5}) H + 0.0015$$

(5.3)

Now applying Herschel Bulkley – law, Flow flux (from equation (5.2), we have

$$\frac{27 \times Q}{2\pi} = \left[\left(\frac{1}{3\eta_m} \right) \left(\frac{P_i - P_f}{Z_f - Z_i} \right) \right]^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$\frac{27 \times 0.01666}{2 \times 3.14} = \left[\left(\frac{1}{3 \times 0.0035} \right) \left(\frac{6140.13 - 2046.71}{0.08} \right) \right]^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$0.071627388 = [4873119.048]^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

On Solving above equation by hit & trial method, We get

$$n = -3.597335$$

Now again from equation (6.11), We have

$$\frac{27 \times Q}{2\pi} = \left[\left(\frac{1}{3\eta_m} \right) \left(\frac{P_i - P_f}{Z_f - Z_i} \right) \right]^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$\text{or } \Delta P = (3\eta_m)(\text{length of vein}) \left[\frac{27Q}{2\pi} \right]^n \left[\frac{6n^3 + 11n^2 + 6n + 1}{26n^3 + 33n^2 + 9n} \right]^n$$

Now putting values of n, Q and length of vein, We have

$$\Delta P = (3\eta_m)(0.08) \left[\frac{27 \times 0.01666}{2 \times 3.14} \right]^{-3.597335} \left[\frac{157.5497688}{815.6929836} \right]^{-3.597335}$$

$$\Delta P = (\eta_m)(0.24)(13142.07601)(370.5861145)$$

$$\Delta P = [(8.51 \times 10^{-5}) H + 0.0015](1168865.013), \eta_m \text{ from equation (5.3)}$$

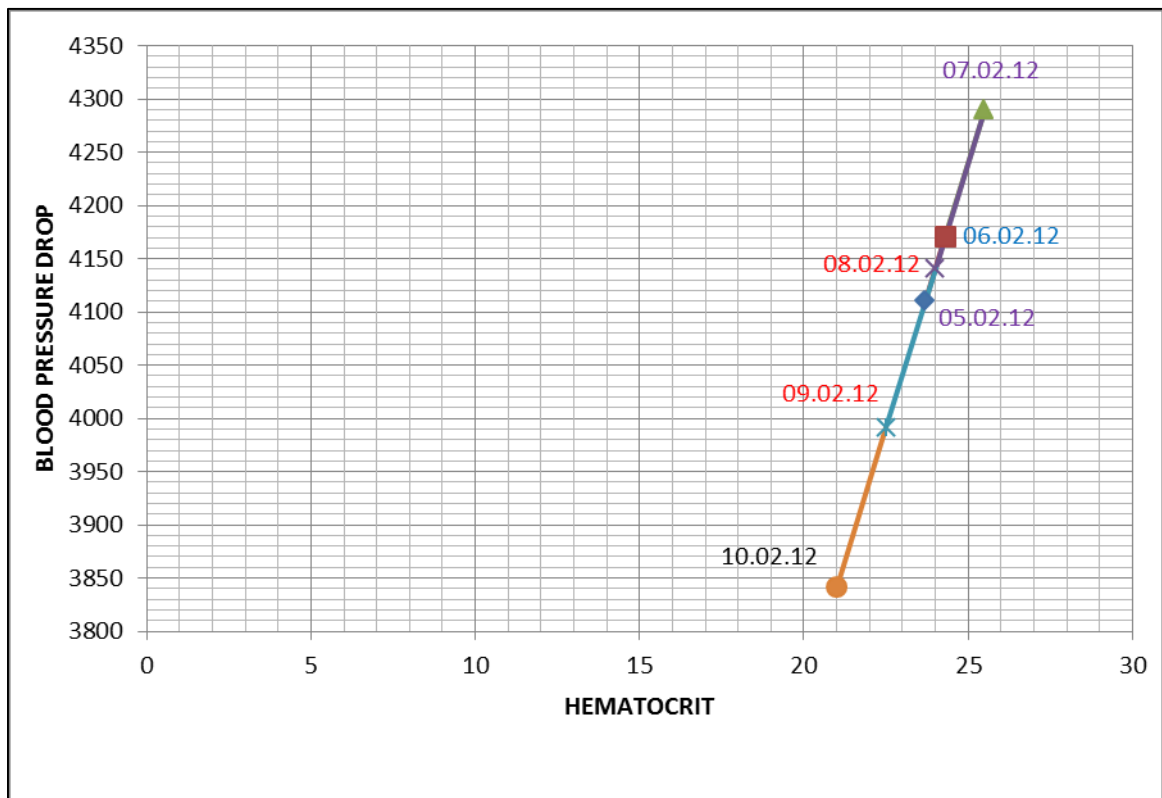
$$\Delta P = (77.6544) H + 1331.2185$$

(5.4)

On Putting values of H in above equation (5.4), We get the following table of blood pressure drop:

Table 2: Blood pressure drop v/s Hematocrit

Date	05.02.12	06.02.12	07.02.12	08.02.12	09.02.12	10.02.12
Hematocrit (H)	23.7	24.3	25.5	24.0	22.5	21.0
B.P. Drop(ΔP)	4110.75	4170.43	4289.79	4140.58	3991.38	3842.18



Graph 1

Observation of Graph

In above graph: (i) we observe that straight line in increasing sense between 05.02.12 to 07.02.12 via 06.02.12 and straight line in decreasing sense between 07.02.12 to 10.02.12 via 08.02.12 and 09.02.12.

Conclusion

Since when blood pressure drop is increased, then we cannot suggest for operation of liver, But when Blood pressure drop is decreased then operation of liver is suggested because hematocrit is directly proportional to blood pressure drop. Above data are not taken during operation, but it is possible according to nature of graph between 07.02.12 to 10.02.12 successful operation is suggested otherwise not.

Remark

If this would have been possible to get blood Pressure on the particular tissue (Liver) during operation of Hepatitis B patient then the graph between blood pressure drop and hematocrit will show more accurate information about operation. That is suppose that the graph shows the decreasing sense meaning thereby the blood pressure drop that is fluctuation in blood pressure is decreasing with decrease of hematocrit. In this situation /duration the successful operation is suggested.

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